MODELING DEPENDENCE ACROSS STOCK MARKETS USING COPULAS

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To Jessy...my love, Arlene, family and friends for their love, encouragement, and support.

.... There is always enough because Christ died and resurrected, thus the hungry are always fed. For God's glory!!!

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Abstract

An important issue in multivariate statistical modeling is the choice of the appropriate dependence measure. Correlation has many pitfalls as it is associated with the elliptical distributions assumption of normality which fails in the presence of extreme endpoints either in marginals or in higher dimensions. Copulas offer an alternative measure of dependence which overcomes the limitations of correlation, and they also determine the type of dependence whether it is linear, upper tail or lower tail. This research serves to explore the appropriateness of copulas in modeling bivariate dependence amongst five SADC stock markets with an objective of assessing the effectiveness of regional integration. Archimedean copulas, due to their desirable properties, were examined using both parametric and non-parametric techniques. Non-parametric estimation gave profound results signifying the appropriateness of the Gumbel copula in dependence modeling which indicated that investors had chances of portfolio diversification across the region as the markets were prone to booming together.

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Chapter 1

Introduction

1.1 Background of the study

Technological advancement and the globalization phenomena have tremendously improved world-wide accessibility in spite of geographical impediments thus influencing social, political and economic reforms on unprecedented levels. The realization of possible mutual benefits among countries in association has led to the formation of world bodies such as the United Nations (UN), Southern African Development Community (SADC), European Union (EU) and the Commonwealth not only for political reasons but also for economic ones. The latter brings into limelight the need to comprehend dependence which is at the forefront of most economic interactions.

The concept of dependence across financial markets has come under considerable study as it is vital in decision-making involving risky assets and varying economic platforms. Multivariate analysis offers an array of methods for studying dependence and these include regression, the principal component analysis, and factor analysis. The most widely used techniques are the joint distributions, the computation of conditional correlations and the copula models.

The multivariate normal distribution is assumed on most given datasets for the joint distributions option, unfortunately, the normality assumption upon which it hinges is of minimal significance for commodity price data, which are known to be leptokurtic. DeCarlo (1997) acknowledges that kurtosis is actually more influenced by measurements in the tails of the distribution than those in the center of it and priorly Hopkins and Weeks (1990) had asserted that skewed data are always leptokurtic. This generates an interest not only in the nature of the measures of dispersion exhibited by the data but also their extent of dispersion as we seek to align our data under study in relation to this consolidated literature as most of this has been concluded on data from developed countries.

Linear correlation coefficient is the key statistic in joint normal distribution and Embrechts *et al* (2002) illustrated the pitfalls and limitations of using it in studying dependence. Most of our study of association has been pinned on the linearity assumption thus restricting us in explaining profound relations in Econometrics. Linear correlation is only a suitable measure of dependence in elliptical distributions, which include the normal distribution and its mixture distributions, outside of which it leads to a number of fallacies. According to Cont (2001), many commodity prices and financial time series display the distributional property of heavy tails, and linear correlation is not defined for some heavy-tailed distributions.

The limitation of the computation of conditional correlations alternative is in that the correlations so obtained have been found to be insufficiently informative in the presence of asymmetric dependence (*correlation breakdown*) i.e. the tendency of the correlations between stock indexes to be dependent on the prevailing direction of the markets, as was envisaged by Boyer, Gibson and Loretan (1999).

The copula model is invariant under increasing and continuous transformations, which is a useful property as transformations are widely used in Economics and Finance. The probability integral transformation is central to the defining of copulas thus confirms their appropriateness in the study of dependence as it helps in the explanation of the level of dependence irregardless of the distribution function of the datasets.

Copulas are also used in the defining of non-parametric measures of association such as the Kendall's tau τ, Spearman's rho ρ as well as the Schweizer and Wolff's sigma σ, which are useful in the parameter estimation of copula models. The copula has the ability to capture tail dependence i.e. a measure of the probability that both variables under investigation are in their extremes which is an important concept in many financial applications. Copulas are of great significance in research work in the fields of Economics and Finance as they account not only for the degree of dependence but also for the dependence structure.

1.2 Motivation of the study

Dependence among explanatory variables in assessing any phenomenon of interest, exemplified by multicollinearity in Regression analysis, autocorrelation in Time Series analysis and co-integration in Econometrics limits the scope of statistical modeling in many research fields. The advent of copula models (*also known as dependence models*) has generated interest in advanced statistical applications especially in Economics and Finance. The emergence and opening up of financial markets in the developing countries with vast potentials of growth especially at regional level, in particular in the SADC region has motivated me to engage in this study.

1.3 Aims and Objectives

The aim of this study is to model dependence across a couple of stock markets within the SADC region using copulas so as to ascertain the extent of mutual benefit they can derive from each other thus spearheading the formation of a regional common market.

The main objectives are:

1. To assess the nature of the stock market index data (*normality, skewness and kurtosis*).

2. To obtain the general movement patterns exhibited across the stock markets.

3. To determine the marginal distributions of the individual stock markets using parametric techniques.

4. To select the appropriate copula models which explain the dependence for all pairs of stock markets based on the computation of their tail dependence.

5. To use maximum likelihood estimation (ML) to estimate the parameters of the proposed copula models.

6. To obtain the joint distribution functions of the pairs of stock markets from the proposed copula models by invoking Sklar's Theorem.

1.4 Significance of the study

Understanding dependence is helpful in empirical finance on portfolio choice and allocation, in particular for regional investors (*portfolio optimization for alternative investments*). It also offers vital information to investors on hedging against risk and in comprehending the variations in stock commodity pricing across counters (*countries*). Copula models are useful in the determination and explanation of the extreme correlations across stock markets under varying economic policies and other external factors.

Advocacy for a common currency and free trade have long been heard in the economic corridors of the SADC states, but unless a better understanding of the dependence structures amongst them is upheld this will always remain a myth. Knowledge of the dependence structures help in stimulating regional economic stability endeavors through the implementation of economic reforms that avert or seek to resolve crises with minimal consequences on trade balance. Copula modeling offers an insight into the possibility and lucrativeness of setting up of the SADC common market in the mold of Common Market of Eastern and Southern Africa (COMESA) or the Preferential Trade Area (PTA).

1.5 Project Layout

This project is organized as follows: Chapter 1 is the introduction which consists of the background to the study, motivation, aims and objectives, and significance of the study. Chapter 2 reviews some basic concepts about stock markets, copula theory, shortcomings of the linear correlation coefficient in studying dependence, profiles of the stock markets under study and literature on handling missing data. The methodology is contained in Chapter 3 which consists of an explanation of the data, the software used, hypothesis tests carried out, the determination of the individual stock markets marginal distributions procedure and the estimation of the proposed copula models' parameters using Maximum Likelihood Estimation. Chapter 4 gives the results and their analysis and Chapter 5 consists of the conclusions and the recommendations from the study.

Chapter 2

LITERATURE REVIEW

Copula modeling of dependence was first applied in finance by Embrechts *et al.*(1999) on their working paper on risk management. A host of other researchers have popularized this concept in the areas of credit risk, multivariate option pricing, portfolio and market value-at-risk (VaR) computations, contagion, tail dependence and time varying dependence of multivariate time series.

Cherubini and Luciano (2001), Embrechts et al. (2003), and Embrechts and Hoing (2006) studied the value-at-risk of portfolios using copula methods due to the presence of non-normal dependence. Longin and Solnik (2001) applied a Gumbel copula across international equity markets to determine their extreme correlations. Marshal and Zeevi (2002) tested the Gaussian assumption in financial markets through the estimation of the degrees of freedom in a t copula. Derivative pricing using copulas was considered by Cherubini et al. (2004) due to the impact of non-normal dependence on pricing and trading.

Ling Hu (2004) used a mixed copula model (*capturing all forms of dependence levels*) to summarize the dependence structure in several major stock markets. The model consisted of a Gaussian copula, Gumbel and Gumbel survival copula, with the latter catering for asymmetric tail dependence. The model was estimated using a two-stage semi-parametric way which is robust and has freedom from specific errors from the marginal distributions. Maximum Likelihood technique was utilized for parameter estimation and the model obtained was useful in the building up of the joint distributions through the utilization of Sklar's Theorem. Presence of conditional heteroscedasticity in the financial data restrained the model results' significance hence a recommendation to initially filter the empirical data was made.

Financial contagion is a phenomenon whereby crises occurring in one market leads to considerable problems in other markets beyond expectations, as exemplified by the Asian markets crush of 1997. Rodriguez (2005) offered the first application of copulas to contagion, using a Markov switching copula model with a focus on the levels and changes in dependence.

Bartram *et al.* (2006) employed a time-varying conditional copula to study financial market integration amongst seventeen European stock market index. Patton (2006) focused on financial time series applications of copulas with an allowance for time-variation. This gave an extension of Sklar's Theorem to Time series data on condition that the information set had to be similar for all marginal distributions in the copula. He used the Inference Function Method (IFM) which consists firstly of parameter estimation for the marginal distributions through maximum likelihood and then parameter estimation for the copula conditional on the estimates computed for the marginal distributions. This technique is computationally easy and offers a description of the cross-sectional dependence between time series and also between observations in a given univariate time series.

Hence it suffices for us to have background knowledge of the stock markets and the theory on copulas for us to understand how dependence across stock markets can be modeled using copulas, and this is given in the proceeding sections.

2.1 Stock exchanges/markets

A stock exchange (*organized)* is a corporate or a mutual corporation established for the trading of investment tools which include company stocks, securities and derivatives. This is where the listing and trading of investment tools occurs under the governance of nationally instituted regulations and laws.

A stock market is one of the most important sources for companies to raise money (*capitalization)* as it allows business to go public and hence raise capital for expansion. Stock markets necessitate the exchange of securities between buyers and sellers, thus providing a marketplace either real or virtual. Real or physical market are where business transactions are carried on a trading floor, where the traders enter into verbal bids and offer simultaneously, usually by the open-cry system. On the other hand, virtual markets are stock markets where trade is done electronically through the matching of bids and offers thus much of the activity is conducted by proxies, i.e. individuals or corporates carrying business on behalf of others.

The other functions of stock markets are that they also provide real-time trading information and protection on the listed securities thereby facilitating fair prices determination. Other than organized stock markets, we also have over-the-counter (OTC) stock markets, where stocks not listed on the organized stock markets are traded and there are no trading floors.

The chief proponent of the stock markets are the participants, who range from small individuals to large hedge fund traders, who, over time have become more institutionalized due to the greater emphasis on safety (*security).* Some of the institutions which have become the principal participants are insurance companies, pension funds, mutual funds, hedge and investor groups as well as banks. Trade is carried out mainly by stock-brokers on behalf of the traders, who also act as financial advisers. The stock market acts as a clearinghouse which eliminates the risk to any buyer or seller in case of a counterpart defaulting on a transaction. Stock owners are issued with certificates and they are not creditors but shareholders and they possess considerable privileges in the running of the corporates they would have bought shares in.

Membership of the markets consists of commission brokers, floor brokers, floor traders and specialists including underwriters, who not only create the market but also facilitate the market for security issues as well as determine prices as they perceive the supply and demand patterns for particular securities. A variety of orders are traded ranging from market orders, limit orders, stop orders and stop-limit orders. Investors specify the period for which orders ought to remain in force (*time period for orders)* and most stock markets recognize two types of such orders. The day order, which is active only for the day on which an order is entered under which market orders fall. The other one is called the open order, popularly termed the Good Till Canceled (GTC), which remains active until either a cancellation is effected by the seller or an execution occurs and it is used mainly in conjunction with limit orders.

2.2 The behavior of stock markets

Business on a stock market thrives upon a large volume of sales and a narrow range between the offer and the bid prices. A well functioning market insists upon rapid execution of orders and confirmed prices offer a fair-market appraisal of a stock. The stock market is characteristically emotional, hence stock prices tend to be volatile though within reach of a generally accepted value (*the intrinsic value*) of a stock. In as much as the Efficient Market Hypothesis (EMH) states that changes in fundamental factors such as dividends should affect share prices, in cases of crashes, no visible factors can explain the price movements other than psychological ones. The overall behavior of a stock market is either bullish or bearish.

A bull market is characterized by a lengthy period of generally accelerating prices/indexes. These can be attributed to optimism, euphoria, investor confidence and expectations that strong results will continue. The number of shares traded is high and also the number of companies entering the market show that the market is confident.

A bear market shows a lack of confidence and it is characterized by a long term downtrend with lower intermediate lows interrupted with lower intermediate highs. Volumes traded are stagnant, indices fall and prices hover at the same level and then plummet.

Psychological effects and speculation play a vital role in market activities hence making it impossible to determine when trends in the market change. A single market can swing from bullish to bearish mood, hence epitomizing the levels of risk involved in investing in shares.

2.3 Stock Market Index

A stock market index is a statistical measure of the changes in a portfolio of stocks representing a portion of the overall market. It represents the characteristics of its component stocks, all of which bear some commonality such as belonging to the same industry or trading on the same stock exchange. A wide range of such indices are available of which the formidable broad-based index represent the performance of a whole stock market thereby reflecting the investor's views on the state of the economy. Ideally, a change in the price of an index represents an exactly proportional change in the stocks included in the index. Weighted index include the price-weighted index and the market-share weighted index where price is determined relative to the number of shares on offer.

The capitalization-weighted index is of great interest here as it takes into consideration the size of the company and hence the contribution of the stock to the index.

The value of the index at time t is the sum of the market capitalization of all index constituents at that time, divided by the sum of the market capitalization of all index constituents at the base date (*for the African financial Markets research group, the base date is* 31^{st} *December 1998*).

$$Index_t = \frac{\sum_{i=1}^{n} Price_{i,t} * Shares_{i,t} * FX_t}{d} \tag{2.3.1}$$

where

Price = Share price

Shares = number of outstanding shares

FX_t = the $US\$$ exchange rate for the home country currency

t = time

i = individual index constituent member

d = the divisor which has been chosen to fix the value of the index at the starting date of 31^{st} December 1998, equal to 1000. The divisor is adjusted to reflect changes in index constituents or capitalization events, to avoid any distortion to the index.

2.4 Investing in stock indices

Singh (1997) concluded that stock markets are expected to enhance economic growth as they offer a boost to the domestic savings, whilst Kenny and Moss (1988) had earlier on postulated that they enhance the operations of the domestic financial system in general and the capital market in particular. The table below offers some of the notable investors in stock indexes and the kind of participation they are involved in.

Table 2.1: Investment in stock indexes

Type of financial institution	Participation in stock markets
Commercial Banks	They issue stocks to boost their capital bases
Savings Banks	Invest in stocks for their investment portfolios
Insurance Companies	Invest premiums in the stock markets
Pension Funds	Invest a part of pension funds in the stock markets
Individuals	Invest their savings in shares to keep them afloat for future use

Consequently, it must be observed that investment in stock indexes does not offer a guarantee on financial prospects due to the unpredictable bullish and bearish behaviors exhibited by the stock markets.

2.5 Globalization of stock markets

Firms in need of funds can consider foreign markets, and investors can purchase foreign stocks, in line with portfolio diversification and optimization. Research has demonstrated that investors in stocks can benefit by diversifying internationally as this is helpful in risk management and has the added advantage of spreading out risk through risk sharing. The need for enhancement of global image also is a profound reason for globalization of stock markets. The proliferation of new financial products especially in the derivatives sector of the stock markets has increased the volume of business due to their high rewards, hence potential investors strategically have become risk-averse beyond borders.

Historically, barriers were there which stalled international stock trading due to technological and socio-political gulfs but have now been eased as outlined below.

1. Reduction in transaction costs

   Many countries have consolidated their stock markets through computerization and enhanced telecommunications thus increasing efficiency and reducing transaction costs. The European markets use an extensive cross-listing (*Eurolist*), so that investors in a given European country can easily purchase stocks of companies based in other European nations. Heightened competition has led to the forging of strategic alliances, mergers and consolidations among stock markets with a view to lowering trading costs as well as to offer investors a variety of financial securities.

2. Reduction in information costs

   The advent of the Internet has made valuable information to be available for investors to make informed decisions without having to purchase information. The accounting standards across countries are also being made uniform for ease of financial data interpretation and comparison, and the constant updates of exchange rates and worldwide news avails valuable information for investment across nations. Bilateral agreements and decision by world governing bodies such as the G-8 on investment in the developing nations offers security and confidence in investors at minimal information costs.

3. Reduction in exchange rate risk

   Advocacy for a single or common denomination of trading as in the case of the Euro, hedge against this risk. Alternatively, agreements are being made to agree on using stable currencies such as the $US\$$ in trading blocs such as the Middle East for the oil trade, with regulations on trading conditions can promote globalization of stock markets.

2.6 Challenges and solutions to developing stock markets

Markets in developing nations offer a high platform for investment opportunity as they are still in their prime of development and their colonial ties offer potential marketability. In spite of these possible advantages, developing markets are faced by a couple of challenges.

The incompetent data storage and management restricts the amount of information upon which investors can make decisions on, especially the financial data, hence much decisions and trading are done in accordance to speculations and rumors. The markets are small in size and also highly volatile due to the small number of shares for other firms thus large volumes of trade tend to jolt the equilibrium prices more frequently, thus making them to be susceptible to manipulation by large traders. Insider trading is also more pronounced because it is difficult to enforce rules and regulations against it. The President of the Asian development Bank in his speech at the ADB Institute Symposium in Tokyo, Japan on the 8^{th} of February 2008, highlighted the impact of global financial linkages on market volatility. The projected slowing down of the US economic growth rate to 1.5% during the year 2008 would affect Asian markets' performance due to the US's significant influence on the global business cycles.

Prevalent political and social upheavals tend to correspond to the occurrence of panic trading, which increases the risk of investors not in tandem with the perceived benefits from such risk. The internal strife in Cote d'Ivoire, is a nation to note as this has caused potential investors to refrain from investing there. Trading infrastructure is lagging on African markets as most operations are manually accomplished, albeit, Singh (1999) envisioned a derailment in their developments due to poor financial structures. An over-concentration of stock market activities on equities limits the variety of products to which investors will be exposed hence their investment options are narrowed.

The highly volatile monetary policies in developing markets also have a bearing on the behavior of stock markets, as low liquidity tend to lessen the investors confidence on the securities on offer. This can be attributed to the low or absence of a well-developed domestic investor base as evidenced by Swaziland's 0.02% turnover ratio, which means that busines is too low and thus the market cannot sustain its activities on its own. In general, the free-float on African markets lies within the $10 - 25\%$ range, which is very low by international standards and hence must be revamped considerably, for competence to be upheld. Major economic problems coupled by capital shortages and immense unemployed human resources were at the centre of the constraints to the initial set-up of the stock market in Ethiopia, in as much as it was urgently needed.

Some of the ways for the promotion of stock markets in Africa and developing nations lie in automation, demutualization, encouragement of institutional investors and foreign participation; regional integration and the strengthening of trade regulations. The President of the Asian Development Bank, in his speech, advocated for the pursuit of sound macro-economic policies and that the policymakers should retain formidable efforts for the maintenance of confidence in their region. Cross-linking of stock markets, together with the lifting of exchange controls, can address the problems of small size and non-liquidity as it enhances their integration and simultaneously retaining their national identity. This is the reason of the undertaking of this project as we seek to understand the dependence amongst SADC stock markets for regional integration and competence to be improved.

2.7 Shortcoming of linear correlation in studying dependence

Most studies on dependence have revolved around the linear correlation, due to its simplicity of computation, easy of manipulation under linear operations and its naturalness as a dependence measure in multivariate normal distributions and in general for multivariate spherical and elliptical distributions.

Campbell, Lo and MacKinlay (1997) showed its importance in the theory of finance as envisaged by its use in the Capital Asset Pricing Model (CAPM) and in the Arbitrage Pricing Theory (APT). In spite of these immense utilization, a host of researchers including Embrechts *et al* (1999) have shown its shortcoming in explaining dependence due to the influx of non-linear derivative products which invalidates many of the underlying distributional assumptions as it cannot be extended to non-elliptical distributions. This can be attributed to its sensitivity to outliers, and the fact that it is only invariant under strictly increasing linear transformation. In insurance, these assumptions fail because of the typical skewness and heavy-tailedness of insurance claim data.

The following are some of the noted pitfalls of linear correlation in measuring dependence;

1 Its requirement that variance be defined between the variables of interest does not hold for heavy-tailed distributions such as the bivariate t_v-distribution for $v < 3$, hence non-life actuaries who model business losses with infinite variances can not get vital inference from it. Cont (2001) also showed that many commodity prices and financial time series display the distributional property of heavy tails and hence non-existence of higher moments.

2 Independence of two variables implies no correlation under the normality assumption but zero correlation does not in general imply independence. The normality assumption is of little significance for commodity price data which are famously leptokurtic.

3 Linear correlation is not invariant under non-linear strictly increasing transformations $T : \Re \rightarrow \Re$ i.e. $\rho(T(X), T(Y)) \neq \rho(X, Y)$
where X and Y are the random variables of interest.

4 In elliptical distributions, a joint distribution can be constructed from the given marginal distributions and correlation ρ value, but in general this does not hold as an increase in the data's noise causes the interval of attainable correlation values to shrink.

Faced with these restrictions consideration of other measures of dependence has to be made which include concordance measures, tail dependence, rank correlation and copulas, of which the latter are more pliable with the limitations of linear correlation.

2.8 Theory on copulas

A copula is a multivariate cumulative distribution function defined on an n-dimensional cube such that every marginal distribution is uniform on $[0, 1]$. It is a tool for modeling dependences among several variables as it describes the interrelation within them. Its name is a derivation of the Latin word (*copulare*) which means to join or connect. A copula consists of two parameters of interest, the association parameter, which controls the degree of dependence and the weight parameter which reflects the shape of dependence. Sklar (1959), came up with this function stemming from the concept of the probability integral transformation stated below;

Result 2.1. *If X has a cumulative distribution function $F(.)$ which is continuous, then the random variable $U = F(X)$ has the distribution $U(0, 1)$.*

Copulas have been instrumental in the development of probability metric space and are also useful in the definition of non-parametric measures of dependence.

Notable contributors to its development are Hoeffding, and M. Frechet and G Dall'Aglio (1950s) who studied the bivariate and trivariate distribution functions with given univariate marginals.

A definition of a copula for the bivariate case is given below;

Definition 2.1. *A copula is a function* $C : [0,1]^2 \to [0,1]$ *which satisfies:*

(a) *C is grounded, i.e for every u, v in [0,1],* $C(u,0) = 0 = C(0,v)$*.*

(b) *C is 2-increasing, i.e for every rectangles* $(u_1, u_2) \times (v_1, v_2) \subset [0,1]^2$ *such that* $u_1 \leq u_2$ *and* $v_1 \leq v_2$, $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$*.*

(c) $\forall\ u, v \in [0,1]$, *C(u,1)=u and C(1,v)=v*

(d) *Copulas are Lipschitz (and hence uniformly) continuous and satisfy:* $\mid C(u_2, v_2) - C(u_1, v_1) \mid \leq \mid u_2 - u_1 \mid + \mid v_2 - v_1 \mid, \forall u_1, u_2, v_1, v_2 \in [0,1]$

The intuition behind the copulas' ability to describe the dependence structure is that the marginal distributions are transformed to Uniform distributions which are then used as the reference cases based on the following result on quantile transformations.

Result 2.2. *For a distribution function F, we define its generalized inverse by* $F^{\leftarrow}(y) = inf\{x : F(x) \geq y\}$*.*

Sklar(1959) showed that if H is a joint distribution function for a pair of random variables X and Y with margins $F(x)$ and $G(y)$ respectively, then there exists a copula C such that $H(x,y) = C(F(x), G(y))$ in the theorem named after him, the Sklar's Theorem stated below;

Theorem 2.1. *Let X and Y be random variables with bivariate distribution function H and cumulative distribution functions F and G, respectively. Then there exists a copula C such that*

$$H(x,y) = C(F(x), G(y)) \tag{2.8.2}$$

$\forall x, y \in \Re$*. If F and G are continuous, then C is unique. Otherwise, the copula C is uniquely determined on Ran(F)* $\times$ *Ran(G). Conversely, if C is a copula and F and G are distribution functions, then the function H defined by equation 2.8.2 is a joint distribution function with cumulative distribution functions F and G.*

Two basic copulas W and M were derived from the Sklar's Theorem as the Frechet-Hoeffding bounds for copulas based on the theorem below;

Theorem 2.2. *Consider a copula C and $\forall u_1, u_2 \in [0,1]$, then*

$$W(u_1,u_2) = max(u_1+u_2-1,0) \leq C(u_1,u_2) \leq min(u_1,u_2) = M(u_1,u_2) \quad (2.8.3)$$

2.9 Families and types of copulas

The most important families of copulas are the Archimedean and elliptical copulas, though there are a variety of other copula families in use. Archimedean copulas model events where there is a lot of risk on individual stocks and elliptical copulas model joint movements between variables.

2.9.1 Elliptical copulas

Under the elliptical copulas we have the Gaussian copula which are based on the normality assumption and they usually act as the benchmark for copulas and the Student t-copula from the Student t-distribution, a derivative of the normal distribution. The Gaussian copula is flexible in that it allows for equal degrees of positive and negative dependence by including both the Frechet bounds in its permissible range, hence it is called a *comprehensive* copula. It can be represented as follows;

$$C_\rho^{Ga}(u_1,u_2) = \Phi_\Sigma\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right) \quad (2.9.4)$$

where Σ is a 2×2 covariance matrix, Φ is the cumulative distribution function of a standard normal distribution and Φ_Σ is the cumulative distribution function of a bivariate normal distribution with zero mean and covariance matrix Σ.

The Student t-copula has two dependence parameters, ν (degrees of freedom) and θ (correlation), where ν controls the heaviness of the tails. The t-copula permits symmetric tail dependence and underestimates dependence when there is asymmetric dependence. It can be represented as follows;

$$C^t_{\nu,\Sigma}(u_1, u_2) = t_{\nu,\Sigma}\left(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2)\right) \tag{2.9.5}$$

where Σ is a correlation matrix, t_ν is the cumulative distribution function of the one dimensional t_ν-distribution and $t_{\nu,\Sigma}$ is the cumulative distribution function of the multivariate $t_{\nu,\Sigma}$-distribution.

2.9.2 Archimedean copulas

Archimedean copulas are of the general form;

$$C(u_1, u_2) = \Phi^{-1}\left(\Phi(u_1) + \Phi(u_2)\right) \tag{2.9.6}$$

where Φ is a decreasing function mapping $[0, 1]$ into $[0, \infty]$ and it is called the *generator* of the copula and this is well explained in the following theorem;

Theorem 2.3. *Consider a continuous and strictly decreasing function* $\Phi : [0, 1] :\to [0, \infty]$*, with* $\Phi(1) = 0$*, then*

$$C(u_1, u_2) = \begin{cases} \Phi^{-1}\left(\Phi(u_1) + \Phi(u_2)\right) & if \quad \Phi(u_1) + \Phi(u_2) \leq \Phi(0) \\ 0 & otherwise \end{cases} \tag{2.9.7}$$

is a copula, if and only if Φ is convex.

Archimedean copulas are the most widely used copulas because they offer a great variety of dependence structures, they can be easily constructed, and there are many parametric types of copulas belonging to it. They allow for a stronger dependence between extreme losses (e.g when stock markets collapses) than between extreme gains, hence a favorite for modelling dependence across stock markets. Archimedean copulas consist of the Product, Gumbel, Joe, Clayton, Frank copulas and their mixtures such as the Clayton-Gumbel.

The Product copula characterizes independent variables when the distribution functions are continuous, where the joint distribution is the product of the marginals i.e $H(x, y) = F(x).G(y)$. The copula function for these two variables is given by the following equation;

$$\Pi(u_1, u_2) = u_1 u_2 \tag{2.9.8}$$

The corresponding generator for this copula is given by

$$\phi(t) = -\ln(t) \tag{2.9.9}$$

Gumbel copula was proposed in 1960 and is expressed as follows;

$$C_\delta^{Gu}(u_1, u_2) = \exp\left[-\left((-\ln u_1)^\delta + (-\ln u_2)^\delta\right)^{\frac{1}{\delta}}\right] \tag{2.9.10}$$

where $\delta \in (1, \infty)$. Values of 1 and ∞ correspond to independence and the Frechet upper bound, but this copula does not attain the Frechet lower bound for any value of δ. Gumbel does not allow for negative dependence but exhibits strong right tail dependence and relatively weak left tail dependence. Its generator is given below as;

$$\phi(t) = (-\ln(t))^{\delta} \tag{2.9.11}$$

If outcomes are known to be strongly correlated at high values (*markets boom together*), but less correlated at low values, then the Gumbel copula is an appropriate choice.

The Joe copula is similar to the Gumbel copula in that they both are suitable for modeling asymmetric dependence. The Joe copula is expressed as follows;

$$C_{\theta}^{J}(u_1, u_2) = 1 - \left((1-u_1)^{\theta} + (1-u_2)^{\theta} - (1-u_1)^{\theta}(1-u_2)^{\theta}\right) \tag{2.9.12}$$

and its generator is

$$\phi(t) = -\ln\left[1 - (1-t)^{\theta}\right] \tag{2.9.13}$$

Clayton copulas were proposed in 1978 and are best described by the following equation;

$$C_{\theta}^{Cl}(u_1, u_2) = max\left\{(u_1^{-\theta} + u_2^{-\theta})^{-\frac{1}{\theta}}, 0\right\} \tag{2.9.14}$$

The marginals become independent when $\theta \to 0$ and the copula attains the Frechet upper bound as $\theta \to \infty$ but cannot account for negative dependence. When correlation between two events is strongest in the left tail (*markets crash together*) of their joint distribution, Clayton copula is an appropriate modeling choice. The generator for this copula is given below;

$$\phi(t) = \frac{\left(t^{-\theta} - 1\right)}{\theta} \tag{2.9.15}$$

The other type of Archimedean copulas is the Frank copula shown below which exhibits no tail dependence because of its symmetry.

$$C_{\theta}^{Fr}(u_1, u_2) = -\frac{1}{\theta} \ln \left(\frac{(\exp^{-\theta u_1} - 1)(\exp^{-\theta u_2} - 1)}{\exp^{-\theta} - 1} \right) \tag{2.9.16}$$

for $\theta \in \Re - \{0\}$. Its generator is described by the function;

$$\phi(t) = \ln \left(\frac{\exp^{\theta t} - 1}{\exp^{\theta} - 1} \right) \tag{2.9.17}$$

The Clayton-Gumbel copula is a mixture of the Clayton and Gumbel copulas and it describes asymmetric dependence by its ability to capture dependence in both tails. It is also called the Generalized Clayton copula and its formula is as shown below,

$$C_{\theta,\delta}^{Cl}(u_1, u_2) = max \left\{ \left([(u_1^{-\theta} - 1)^{\delta} + (u_2^{-\theta} - 1)^{\delta}]^{\frac{1}{\delta}} + 1 \right)^{-\frac{1}{\theta}}, 0 \right\} \tag{2.9.18}$$

.

2.10 Copulas and measures of association

Some of the attributes of rank correlation measurements are that they are insensitive to outliers, they are invariant under strictly increasing linear transformations and that they depend only on the copula of the two random variables under investigation. This is significant in the formulation of the underlying copula for modeling dependence, typified by the relationship postulated by Genest and Mackay (1986) between Kendall's τ and Archimedean copulas. Hence the Kendall's τ, can be computed in terms of copulas as follows;

$$\tau_C = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1 \tag{2.10.19}$$

where C is the copula associated with the random variables, say (X,Y).

Spearman's ρ is defined in terms of the copula associated with the random variables as follows;

$$\rho_C = 12\int_0^1\int_0^1 \left(C(u,v) - uv\right) dudv \tag{2.10.20}$$

Shweizer and Wolff's σ can be computed in terms of copulas as follows;

$$\sigma_C = 12\int_0^1\int_0^1 \mid C(u,v) - uv \mid dudv \tag{2.10.21}$$

Genest and Mackay (1986) showed that for Archimedean copulas, Kendall's τ can be expressed in terms of the generator of the copula as follows;

$$\tau = 1 + 4\int_0^1 \frac{\phi(t)}{\phi'(t)} dt \tag{2.10.22}$$

This was verified by Joe (1997) and hence, Kendall's τ in terms of the association parameter θ for some Archimedean copulas are given in the table below;

Table 2.2: Kendall's τ for some Archimedean copulas

Copula	Kendall's τ
Clayton Copula	$\frac{\theta}{2+\theta}$
Gumbel Copula	$1-\frac{1}{\delta}$
Frank Copula	$1-\frac{4}{\theta}+\frac{4}{\theta^2}\int_0^{\theta}\left(\frac{t}{\exp^t-1}\right)dt$
Clayton-Gumbel	$\frac{(2+\theta)^{\delta}-2}{(2+\theta)^{\delta}}$

2.11 Tail dependence

This is a phenomenon which captures asymmetric correlation due to the extreme values contained in multivariate time series data in particular. It is the probability that both variables under investigation, in a bivariate study, are in their extremes. Its existence is a key issue on copula selection for use in modeling dependence. Copulas are able to capture it within the dependence structure, and of particular interest are the coefficients of the lower and upper tail dependence. Archimedean copulas utilizes the L'Hopital's rule in obtaining these coefficients, which is given in the following result,

Result 2.3. *Let c be either a finite number or ∞, if $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = 0$, then* $\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{df(x)}{dg(x)}$

Upper tail dependence means that given large values of u_1, then large values of u_2

are expected. Its coefficient is computed by the following equation;

$$\lambda_U = \lim_{u_1 \to 1^-} \left(\frac{1 - 2u_1 + C(u_1, u_1)}{1 - u_1} \right) \tag{2.11.23}$$

provided that the limit $\lambda_U \in [0, 1]$ exists.

The coefficient for the lower tail dependence is computed using the following equation;

$$\lambda_L = \lim_{u_1 \to 0^+} \left(\frac{C(u_1, u_1)}{u_1} \right) \tag{2.11.24}$$

provided that the limit $\lambda_L \in [0, 1]$ exists.

If $\lambda_U > 0$ or $\lambda_L > 0$, then we say that the two random variables under investigation have either upper tail or lower tail dependence.

Elliptical copulas are symmetric, hence we have $\lambda_L = \lambda_U$, which is zero for the Gaussian copula and greater than zero for the t copula. Presence of tail dependence in data suggests the inappropriateness of the Gaussian copula whilst its asymmetric state annuls the t copula. The expected levels of tail dependence in Archimedean copulas are as tabulated below;

Table 2.3: Tail dependence levels for Archimedean copulas

Copula	Upper tail, λ_U	Lower tail λ_L
Gumbel	$2-2^{-\frac{1}{\delta}}$	0
Joe	0	>0
Frank	0	0
Clayton	0	$2^{-\frac{1}{\theta}}$
Clayton-Gumbel	$2-2^{-\frac{1}{\delta}}$	$2^{-\frac{1}{\delta\theta}}$

2.12 Modeling dependence using copulas

The following are the advantages of using copulas in modeling dependence.

1. Offer an allowance to model both linear and non-linear dependence.
2. There is an arbitrary choice of the marginal distributions.
3. They are capable of modeling extreme data values.
4. They are invariant under increasing and continuous transformations. This property is very useful as transformations are commonly used in economics.

Copulas are widely used in Finance and Economics especially in Insurance and risk management, as well as in Multivariate survival modeling and Bio-informatics. The knowledge of dependence structure among financial securities is important in the sectors of portfolio management, control of risk clustering, pricing and hedging. The normality assumptions fails here, such that investment decisions based on linear correlation could thereby jeopardize business prospectives in the long run.

2.12.1 Fitting copulas to data

A variety of techniques are employed in fitting copulas to data which include the Inversion method, Inference Function Method for marginals (IFM), maximum likelihood method, the Empirical Copula Function (ECF) and estimation from Kendall's τ. In this research the techniques used for fitting copulas to data are the estimation of copula parameters from Kendall's τ and the parametric estimation of copulas.

The former technique is more appealing due to the presence of a relationship between rank correlation coefficients with copulas and the relative ease of the computations involved. This is a non-parametric technique and the relations in Table 2.2 are used to determine the parameters for the Archimedean copulas. Goodness of fit tests are conducted basing on the Akaike Information Criterion, the Bayesian Information Criterion and the log-likelihood statistics. The tail dependence levels are computed using the relations in Table 2.3.

In parametric estimation of copulas, there are two conventional approaches that can be adopted which are the joint estimation and the parametric two-stage estimation techniques. Joint estimation involves the estimation of both the marginal distributions of the variables of interest alongside the copula. This induces misspecification error as estimates of marginal distributions affect those for the copula. For a given copula model, and specified marginal distributions, the copula density is computed from

$$c(u, v; \theta) = \frac{\delta^2 C(u, v; \theta)}{\delta u \delta v} \tag{2.12.25}$$

We obtain the likelihood function for the paired observations in terms of the marginal distributions, marginal densities of the random variables and the copula density. Maximum likelihood technique is then applied to obtain the copula parameter estimates by maximizing the log likelihood function. The limitations in this approach are apparent when both the copula and the marginals exhibit some complicated forms or when there are too many parameters involved.

Two-stage parameter approach, as its name suggests involves two stages, initially the assumption of independence is upheld on the variables of interest and their marginal distributions are determined parametrically based on the log likelihood statistic. These are then treated as nuisance parameters in the proposed copula model and straight away the technique of maximum likelihood estimation is utilized in the estimation of the estimates for the copula's parameters.

The maximum likelihood (ML) method is a parametric technique that can also be employed in fitting copulas to data. Multivariate analysis' focus on the dependence structure requires that the dependence parameter should be margin-free, parametric procedures gives estimates that are margin dependent. Hence, an appropriate alternative would be for us to utilize the semi-parametric canonical maximum likelihood (CML) method in which estimation of the association parameter of the copula is made without assuming any parametric form for the marginal distributions of the financial variables. This has the advantage in that it is a robust approach and that non-specification of the underlying distribution allows our results to be free of the distributional misspecification errors. The empirical cumulative distribution function of a random variable say X (*which is an approximation of the true cumulative distribution function*) is computed by the following formula;

$$F_X(x) = \frac{numberof\ x_t \leq X}{N} \tag{2.12.26}$$

A set of joint observations (X_t, Y_t) of X and Y, any two random variables can be transformed into a set of points (u_t, v_t) in the unit square by defining $u_t = F_X(x_t)$ and $v_t = G_Y(y_t)$ utilizing the Probability Integral transform. CML involves the maximization of the likelihood of these data sets.

This study seeks to utilize copulas in modeling dependence across the stock markets using the non-parametric and the parametric two-stage techniques in five selected nations within the SADC region whose profiles are as given in the following section.

2.13 Profiles of stock markets

This project was conducted on five stock markets within the Southern African Development Community (SADC), with the objective of modeling the dependence amongst themselves in their activities so as to assess the possibility of setting up a strong regional trading bloc. Table 2.4 below, gave an annual US$ returns for these stock markets as at 12^{th} of February 2008.

Table 2.4: Regional bourses Year-to-Date US$ returns: 12.02.08

Stock Exchange	US$ returns
Lusaka Stock Exchange	22%
Malawi Stock Exchange	11%
Botswana Stock exchange	-8%
Johannesburg Stock Exchange	-15%
Zimbabwe Stock Exchange	-50%

The Lusaka stock exchange's high performance can be attributed to the perfomance of Shoprite, the Zambia's Sugar company and the copper mine conglomerates. The persistent power outages and the weakening Rand due to the drying up of securitization market in Europe sheds light into the down spiral of returns on the Botswana and Johannesburg stock exchanges. The Zimbabwe stock exchange has witnessed high correlation with market liquidity with notable deviations during events such as the price blitz of July 2007.

Outlines of the set-up of these stock exchanges and brief descriptions of their performance are given in the proceeding subsections.

2.13.1 Malawi Stock Exchange (MSE)

This was established in March 1995 and opened for business in November 1996 under the watchful eye of the Reserve Bank of Malawi. It is a fully-fledged stock market and its membership consists of individuals and corporate businesses and initially had a single licensed broker. It operates in terms of the Capital Markets Development Act (1990) and the Capital Market Development Regulations (1992) and they utilize a single price auction system. Equities and government securities are traded, and foreign investment is restricted to 5% of issued share capital for a foreign investor; and there are no regulations regarding capital repatriation as long as the investor is registered with the Reserve Bank of Malawi. Information is disseminated through weekly reports as trading occurs during the working week days. Clearing occurs by transaction and settlement occurs on a $T+7$ basis i.e 7 days after the trade and there is no depository.

2.13.2 Lusaka Stock Exchange (LuSE)

This is the stock market for Zambia and it was established in 1994 with help from the World Bank's International Finance Corporation for it to meet the G-36 recommendations for clearing and settlement system design and operations. The Barclays Bank of Zambia and Stanbic Bank Zambia offer global custody services, thus forming the electronic depository facilities constituting the Central Share Depository. Activities in the market are controlled by the 1993 Securities Act which necessitated the setting up of the Securities and Exchange Commission (SEC). Zambia's privatization process influenced most of the listings on the LuSE especially in the mining sector. Trade is mainly in equity securities and government bonds, and foreign investment is not restricted and there are six stockbroking firms active on the exchange.

2.13.3 Johannesburg Stock Exchange (JSE)

This was established primarily to raise capital for the mining industry thereby enhancing job opportunities and wealth creation. It is the largest stock market in Africa and is among the top ten largest markets in the world. It was demutualized in July 2005 and it lists shares on two separate markets, the Mainboard and AltX (*smaller, black-owned enterprises*). Electronic trading is conducted on stocks, bonds and derivatives, using the JET system (Johannesburg Equities Trading) which is order-based, whereby transactions are automatically executed when matching buy and sell prices are found. It has trade links with the Namibian Stock Exchange which was effected in 1998 through a telecommunication link to the JET system. Membership is open to both corporate bodies and foreigners and the main challenge is that of insider trading.

2.13.4 Botswana Stock Exchange (BSE)

The Botswana Development Corporation (BDC) spearheaded public participation in the economy of Botswana through Sechaba Investment Trust which initiated the Botswana share market in June 1989. Secondary market trading was facilitated by the Stockbrokers Botswana Limited, whose fees and commissions were determined in line with rates from the Zimbabwe Stock Exchange. An Interim Stock Exchange Committee was set up in October 1990 to encourage foreign investment and in 1994 a Stock Exchange Act was passed which formally established the Botswana Stock Exchange. There is one index, which includes all listings, the Botswana Share Market Index. Private investors take up at most 10% of the total market capitalization and foreign investment is restricted to 10% of the issued capital ofa publicly owned company and funds repatriation is limited. It is generally a high performer.

2.13.5 Zimbabwe Stock Exchange (ZSE)

This was initially established in 1896 by the Pioneer Column and later frozen due to the World Wars. The Zimbabwe Stock Exchange Act was put in place in 1974 and re-affirmed at independence in 1980 as the Stock Exchange Act, which was later ammended in 1996's Zimbabwe Stock Exchange Act: Chapter 24:18. It consists of three indexes, the Zimbabwe industrial index, the Zimbabwe Mining index and Natfoods. There are more than 65 listed securities and corporates are not allowed to entitle more than 40% of their ownership to foreigners.

Chapter 3

METHODOLOGY

3.1 The Data

Real life data was used in this research and it was obtained from African Financial Markets. This database covers and maintains financial markets data for a couple of African nations on macro-economics and micro-economics variables.

The data under investigation consisted of stock index from five stock markets within the SADC region, Botswana, Malawi, South Africa, Zambia and Zimbabwe. The time period of study covered the trading dates from the 13^{th} of March 2003 to the 4^{th} of June 2007 consisting of 565 trading days, a period of about four years. Improper data management and warehousing as well as varying technological impediments and national policies, accounted for the missing data values, and these were considered as randomly missing. However, the sampled data from the source database was sufficient for robust inference to be made without loss of generality on the relationships amongst the stock markets under study.

3.2 Nature of Data and general movement patterns across stock markets

Financial returns data is widely acknowledged to be skewed and leptokurtic, this had to be ascertained for the data under study using the coefficients of skewness and kurtosis. Linear correlation coefficients were computed to assess the degree of dependence as well as the general movement patterns amongst the five stock markets. Dependence analysis for this research was limited to pairwise investigations amongst the five stock markets under study. Time series plots of the five stock markets index data were also considered to investigate the movement patterns with time of the stock markets which gives an insight in the type of copula to be used in modeling the dependence pattern amongst the stock markets.

3.3 Normality test and detection of the presence of tail dependence

Normality tests were carried out for the hypothesis asserting that the stock markets index data followed the normal distribution so as to assess the validity of the Gaussian assumption and hence ascertain the appropriateness of the Gaussian copula in modeling this data. The techniques used for normality test where the Jarque-Bera, and Kolmogorov-Smirnov tests respectively. The Jarque-Bera statistic is a goodness-of-fit measure of the departure from normality, based on the sample kurtosis and skewness given by

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{3.3.1}$$

where n is the number of observations, S is the sample skewness and K is the sample kurtosis.

It has an asymptotic chi-square distribution with two degrees of freedom, hence is used in testing whether the sampled data are from populations exhibiting the normal distribution.

Existence of tail dependence is a key issue in the choosing of the copula to be used for modeling the dependence structure between stock markets. Kurtosis is a measurement of the peakedness and tail heaviness of a distribution relative to that of the normal distribution, hence its measurements were used to assess the presence or absence of fat tails dependence. The determination of the underlying marginal distributions also gave a hint on the absence or presence of heavy tails.

3.4 Determination of the marginal distributions of the stock markets

Parametric estimation for the determination of the marginal distributions underlying each of the five stock markets were conducted utilizing the statistical package

MINITAB. The best fit model was selected based on the log-likelihood statistic and the Akaike Information Criterion (AIC). These criteria identify an optimum model for the description of the stock market data from a class of competing models. The log-likelihood statistic is given by the following equation;

$$\eta = -2\sum_{i=1}^{n} log(\frac{g(x_i \mid \hat{\theta_k})}{g(x_i \mid \hat{\theta_L})}) \tag{3.4.2}$$

where $\hat{\theta_k}$ and $\hat{\theta_L}$ are the estimated parameters of the fitted and the true models respectively.

The Akaike Information Criterion statistic is computed using the following equation;

$$AIC = \eta + 2k \tag{3.4.3}$$

where η is equivalent to the log-likelihood statistic, and k is the number of fitted parameters in the model. The first term on the right-hand-side of the above equation measures the lack-of-fit of the chosen model, while the second term measures the increasing unreliability of the chosen model due to the increased number of model parameters. The best approximating model is the one which achieves the minimum AIC in the class of competing models. This is a fairly accurate test as it is based on the maximum likelihood function, which is asymptotically effective and unbiased.

3.5 Copula selection and parameter estimation

An assumption of the copula family used for modeling was made, in this case, Archimedean copulas. Non-parametric estimation utilizes the relationship between the Archimedean copulas and the Kendall's tau. Considering each pair of stock markets, we assume that their joint distribution function H has an associated Archimedean copula C_ϕ. Genest *et al*'s algorithm for the relations in Table 2.2 is as follows;

1. Compute Kendall's τ coefficients.

2. Construct a non-parameter estimate of $K(z)$, which relates the copula estimates with the generator functions. Firstly, we let $Z_i = H(X_i, Y_i)$ be an unobserved random variable with distribution function

$$K(z) = Prob\,[Z_1 \leq z] = z - \frac{\phi(z)}{\phi'(z)} \tag{3.5.4}$$

Let $Z_i = \frac{1}{n-1}\sum_{j=1}^{n}(if[X_j < X_i \text{ and } Y_j < Y_i, 1, 0])$, for $i, j = 1, ..., n$, then

$K(z) = \frac{1}{n}\sum_{i=1}^{n}(if\,[Z_i \leq z, 1, 0])$,

where $function[\text{condition}, 1, 0]$ gives 1 if condition holds and 0 otherwise. The respective $K(x, \theta)$ functions for the Archimedean copulas are as given in the table below;

Table 3.1: Distribution function $K(x, \theta)$ for some Archimedean copulas

Copula	$K(x,\theta)$
Clayton copula	$x - \frac{x^{\theta+1}-x}{\theta}$
Gumbel Copula	$x - \frac{xlnx}{\theta}$
Frank Copula	$x - \frac{ln\left(\frac{exp^{-\theta x}-1}{exp^{-\theta}-1}\right)}{\theta}(exp^{\theta x}-1)$

3. Use relations in Table 2.2 to compute the copula parameter and those in Table 2.3 to compute their respective tail dependence coefficients.

Goodness-of-fit tests were carried out using the tail dependence analysis.

The parametric estimation of the parameters of the particular copula were made using Maximum Likelihood Method in the R statistical package, and the best fit copula models were determined based on the Bayesian Information Criterion (BIC) statistic. This criterion is based on the maximum likelihood function and is an improvement on the log-likelihood statistic, η as it takes into consideration the influence of the sample size n in assessing the adequacy of the model. It is computed using the following equation;

$$BIC = \eta + 2 * ln(n) \tag{3.5.5}$$

where n is the sample size.

3.6 Comparison of estimates

Based on the goodness-of-fit test results, a comparison of the best fit copula models' estimates were then made to highlight the difference between the non-parametric and the parametric technique with a view of recommending the semi-parametric technique in future researches based on these data. Computations of the tail dependence values also gave a comprehensive comparison of the appropriateness of the postulated estimates as well as the extent of heavy-tailedness in the respective pairs of stock markets. The latter also gives a link to the nature of co-movements between the pairs of stock markets.

3.7 Determination of the joint distribution functions for the stock markets

Sklar's theorem relates the copula model to the joint distribution of any two random variables with specified marginal distributions by utilizing the Probability Integral Transformation or the Laplace transformation. For each pair of stock markets and their proposed best fit copula model, the joint distribution functions were estimated using the concepts of Sklar's theorem.

3.8 Software used

The following statistical packages, MINITAB and R were used for the analysis of data in this research whilst Latex was used for all the typing.

Chapter 4

ANALYSIS AND RESULTS

The index data covering the trading period from March 2003 to June 2007 for the five stock markets was used for this analysis. This consisted of 565 data points for each market, constituting trading dates on which all the five markets were simultaneously operational. The statistical packages used for this analysis were MINITAB and R.

4.1 Correlation analysis

Scatterplot matrix of all the possible pairs of the stock markets indicated the presence of dependence amongst them, justifying the need to conduct a correlation analysis. The correlation matrix between the five markets are given in Table 4.1. It can be seen that the dependence between South Africa and Zambia markets are the strongest, and that between South Africa and Zimbabwe markets are the weakest among the ten pairs. Excluding Zimbabwe, all other markets were strongly dependent between themselves suggesting that they had a high tendency of growing together and also that bullish characteristics are more prevalent in them. All other markets were relatively weakly dependent on the Zimbabwe market, which could be a hindrance on the regional integration efforts being spearheaded.

Table 4.1: The linear correlation coefficients across stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	0.9815			
South Africa	0.9487	0.9621		
Zambia	0.9502	0.9605	0.9851	
Zimbabwe	0.7358	0.6933	0.5889	0.6241

The Kendall τ correlation coefficients were computed for the determination of the non-parametric estimates of the Archimedean copula estimates and are given in Table 4.2. Their chief characteristics of insensitivity to presence of outliers as well as their invariance under strictly increasing linear transformations can be attributed to the high Kendall τ correlation coefficients amongst the five stock markets in Table 4.2. This also tallies well with our argument that an alternative technique must be considered for the analysis of the dependence structure among the five markets under investigation. The coefficients for Zimbabwe with respect to all other markets are considerably higher relative to the ones exhibited in Table 4.2, thus indicating presence of outliers in the Zimbabwe stock index data.

Table 4.2: The Kendall τ correlation coefficients across stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	0.9649			
South Africa	0.9101	0.9121		
Zambia	0.9441	0.9599	0.9044	
Zimbabwe	0.9003	0.9066	0.8704	0.9005

4.2 Nature of the data and general movement patterns across the stock markets

The skewness statistics of all the five stock markets were positive, while the kurtosis statistics were all negative except for the Zimbabwe Stock Market as shown in Table 4.3 below. Their magnitudes indicates that all the series had short and thin right tails, i.e mostly were non-peaked whilst the Zimbabwean one was peaked. These indicate that the data were skewed and hence modeling them with elliptic distributions would not be correct. The positive skewness statistics also reveals that the markets are experiencing mainly a bull run behavior signifying that a hive of brisk business activities is being conducted there.

Table 4.3: The skewness and kurtosis statistics for the five stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia	Zimbabwe
Skewness statistic	0.9783	0.7551	0.2577	0.2881	3.9728
Kurtosis statistic	-0.2805	-0.7062	-1.2764	-1.0046	18.8857

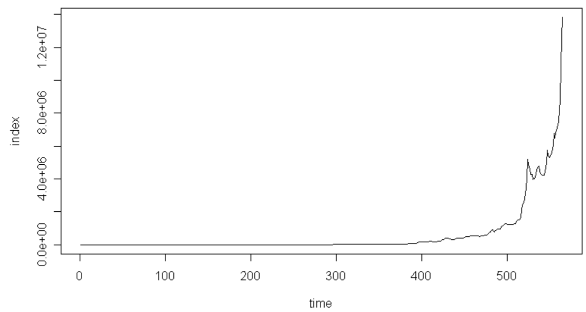

Figure 4.1: Time Series plot for Zimbabwe stock market index data

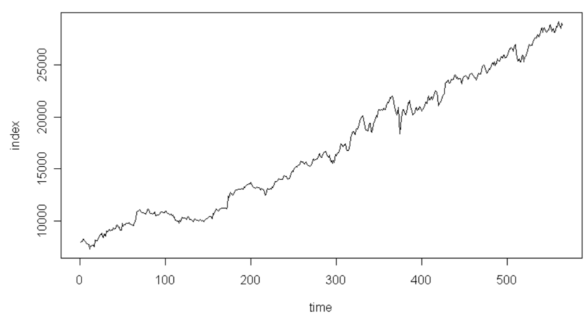

Figure 4.2: Time Series plot for South Africa stock market index data

Time series plots for the index data for the stock markets exhibited upward trends in most of them, typified by the plot for the South African market in Figure 4.2, and in particular that for Zimbabwe tended more to be exponentially curved.

Generally, it can be said that the index data were increasing progressively with the passage of time as evidenced by plots in Figure 4.1 and Figure 4.2 above.

4.2.1 Normality Testing

Jarque-Bera test statistics were computed for each stock market to validate the assumption that the series' exhibited the normal distribution and the decision was made at the significance level of 5%, taking into consideration that the Jarque-Bera statistic is known to asymptotically follow the chi-squared distribution with 2 degrees of freedom at the specified level of significance. Hence, the computed Jarque-Bera statistics for the five stock markets tabulated below were compared against $\chi^2_2(0.05) = 5.99$.

Table 4.4: The Jarque-Bera test for normality for the five stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia	Zimbabwe
Jarque Bera	343.47	377.06	436.77	385.35	7 427.11

The Jarque-Bera statistics were significantly large, hence we rejected the null hypothesis that the stock markets data exhibited the normal distribution. The normality assumption was also rejected using a non-parametric test for normality and the results were as shown below.

Table 4.5: The Kolmogorov-Smirnov test for normality for the five stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia	Zimbabwe
p-value	$< 2.2 * 10^{-16}$	$< 2.2 * 10^{-16}$	$3.286 * 10^{-8}$	$7.71 * 10^{-5}$	$< 2.2 * 10^{-16}$
D	0.199	0.2254	0.1259	0.0948	0.3571

The Kolmogorov-Smirnov test for normality also showed that the series were non-normal. The D statistics indicated that the empirical distribution functions for the series were significantly different from the normal distribution as their values were statistically different from zero. The rejection of the null hypothesis of normality, highlighted either the presence of outliers in the series under study or the exhibition of non-linearity properties in these series.

4.3 Non-parametric estimation of copulas

Three types of Archimedean copulas were estimated non-parametrically based on their relations with the Kendall τ correlation coefficients. Estimates for θ in the Clayton, Gumbel and Frank copula were as tabulated below;

Table 4.6: Estimates of the parameter (θ) for the Clayton copula models for dependence across stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	54.98			
South Africa	20.25	20.75		
Zambia	33.78	47.88	18.92	
Zimbabwe	18.06	19.41	13.43	18.10

Table 4.7: Estimates of the parameter (θ) for the Gumbel copula models for dependence across stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	28.49			
South Africa	11.12	11.38		
Zambia	17.89	24.94	10.46	
Zimbabwe	10.03	10.71	7.72	10.05

It was noted that the parameter estimates of the Clayton copula where almost twice the value of the estimates for the Gumbel copula.

Table 4.8: Estimates of the parameter (θ) for the Frank copula models for dependence across stock markets

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	2.1971			
South Africa	2.1508	2.1524		
Zambia	2.1792	2.1927	2.1461	
Zimbabwe	2.1428	2.1479	2.1188	2.1429

The Frank copula formula for the estimation includes special integrations such as the Riemann Zeta function which can be computed more rigorously using MATLAB, hence an estimate of the integral was used for the computation.

4.4 Tail Dependence Analysis

Tail dependence values were computed at the level of significance of 0.01 for both the lower and upper measurements based on the parameter estimates for the Clayton and Gumbel copulas respectively, from Kendall's τ coefficients.

Table 4.9: Lower and (Upper) tail dependence values for all stock market pairs at $\alpha = 0.01$

Stock Market	Botswana	Malawi	South Africa	Zambia
Malawi	0.9874(0.9989)			
South Africa	0.9663(0.9974)	0.9671(0.9975)		
Zambia	0.9797(0.9983)	0.9856(0.9988)	0.9640(0.9973)	
Zimbabwe	0.9623(0.9972)	0.9649(0.9973)	0.9497(0.9965)	0.9624(0.9972)

Extreme evidence of asymmetry tail dependence were revealed in all the stock market pairs under investigation. Table 4.9 above, revealed that upper tail dependence was slightly more pronounced in all possible stock market pairs relative to the lower tail dependence. This suggests that the stock markets were more susceptible to making significant gains together than making significant loses together.

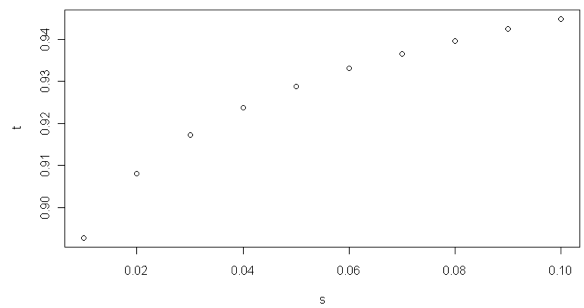

Figure 4.3: Lower tail dependence for the Gumbel copula

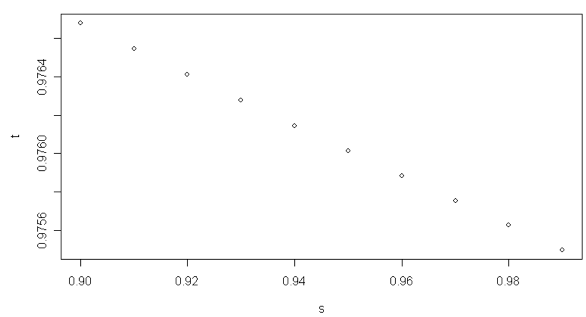

Figure 4.4: Upper tail dependence for the Gumbel copula

Figure 4.3 showed that when s→0, where *s* represents the percentile points of risk, the lower tail dependence tends to zero for the Gumbel copula. The values of upper tail dependence in figure 4.4 were almost constant, as *s* approached one, for all the stock markets thus indicating the appropriateness of the Gumbel copula as the best model from non-parametric copula modeling. Similar analysis for the Clayton copula failed to illustrate their appropriateness as lower tail dependence failed to be almost constant on all stock markets. Hence the best fit copula estimation representation was as given below;

$$\hat{C_\alpha}(u,v) = exp\{-[(-log(u))^\theta + (-log(v))^\theta]^{\frac{1}{\theta}}\} \tag{4.4.1}$$

4.5 Parametric distribution of the stock index data (marginal distributions)

MINITAB was used to determine the most plausible probability distributions that could describe the movement patterns over time of the index data for the stock markets. The Extreme Value (Gumbel) distribution proved to be the optimum model for all the stock markets. This has the following probability density function,

$$f(x) = \frac{1}{\beta}e^{-\frac{(x-\mu)}{\beta}}e^{-e^{-\frac{(x-\mu)}{\beta}}} \tag{4.5.2}$$

and its marginal distribution or cumulative distribution function is given by;

$$F(x) = e^{-e^{-(\frac{x-\mu}{\beta})}} \qquad for -\infty < x < \infty, \quad \beta > 0 \tag{4.5.3}$$

The estimates of the parameters of this model on the stock markets and their corresponding log-likelihood statistics(η) as well as their respective AIC were as tabulated below;

Table 4.10: Parameter estimates for the Extreme Value distribution for the stock markets

Stock Market	Location parameter (μ)	Scale parameter (β)	η	AIC
Botswana	5134.28	1999.55	-5132.17	-5128.17
Malawi	1332.17	777.25	-4604.37	-4600.37
South Africa	20271.30	5998.80	-5782.02	-5778.02
Zambia	1490.69	595.50	-4468.78	-4464.78
Zimbabwe	1644817	3160677	-9207.29	-9203.29

All the stock markets index data exhibited the Extreme Value distribution as their marginal distribution. Copula models have an advantage of the arbitrariness in the choice of the underlying marginal distribution for the data under investigation, hence these were then used as nuisance parameters in the parametric selection of the most appropriate copula models for the stock market pairs under investigation.

4.6 Parametric selection of copula models for the stock markets

The method of maximum likelihood was utilized in the determination of the best fit Archimedean copula models using the statistical package R. Since all the underlying marginal distribution functions for the stock markets where continuous,according to Sklar's Theorem (2.1), there exist unique copulas that explains the dependence structure for all the pairs of stock markets under investigation. The best fit copula were determined using the Bayesian Information Criterion of which 80% of the fits were best explained by the Clayton copula and 20% were explained by the Gumbel copula, as tabulated below.

Table 4.11: Parameter estimates for the best fitted copulas for the stock markets

Stock Markets	Clayton (θ)	Gumbel (θ)	σ	BIC
Botswana and Malawi		7.14	0.364	-8475.95
Botswana and S. Africa	8.86		0.580	-9932.67
Botswana and Zambia	6.98		0.435	-8662.42
Botswana and Zimbabwe	6.17		0.876	-14002.20
Malawi and S. Africa	9.76		0.658	-9458.34
Malawi and Zambia	9.67		0.60	-8049.11
Malawi and Zimbabwe	3.25		0.416	-13574.23
South Africa and Zambia		9.16	0.487	-9180.36
South Africa and Zimbabwe	3.51		0	-14898.31
Zambia and Zimbabwe	6.88		0.243	-13548.26

The underlying distributions for the stock markets of the Extreme Value (Gumbel) distribution gives the Clayton copula as the most appropriate copula in modeling dependence across these stock markets.

One of the issues in copula modeling is to get a proper understanding of the tail behaivor of the data under investigation, parametric copulas may not be an appropriate tool as the marginal misspecifications on the copula parameter estimation depends on the sign of skewness. For this research the skewness statistics are all positive as shown in Table 4.3 indicating presence of upper tail dependence. The parametric estimation results in Table 4.11 offsets the bias in copula estimation to Clayton copula due to misspecification errors on the underlying distribution functions of the stock indices; while the skewness measurements asserts that the Gumbel copula is more appropriate as they reveals the presence of tail dependence in the data which is a pivotal measure in dependence modeling. The appropriateness of the non-parametric technique's best copula, which is the Gumbel copula is encompassed in the fact that its goodness-of-fit assessment hinges on the behavior of tail dependence in the extreme scenarios.

4.7 Comparison of the estimates

The best fit models based on the non-parametric and parametric estimation techniques were the Gumbel and Clayton copula respectively. Based on the goodness-of-fit techniques employed it was noted that the non-parametric estimation results were more appropriate; due to the fact that tail dependence is a copula property, and its use in goodness-of-fit captures the most significant skew exhibited in the data. Distributional misspecification errors could not be ruled out in the parametric estimation as these yielded marginal dependent estimates, which involved complexity in the form of the marginal distributions.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

This thesis found strong evidence of pairwise, and mutual dependence across the stock markets. The high correlation coefficients amongst the five stock markets, with the exception on the Zimbabwe Stock market, indicated that the regional integration of markets is a feasible venture. The strong levels of concordance amongst the markets supports the fact that the markets tend to boom together with a higher chance/probability than crushing together hence offering a platform for portfolio diversification across markets for investors.

The stock markets data exhibited non-normalcy and it was heavily skewed to the right. Most stock markets index data were non-peaked with the exception of Zimbabwe which was peaked. The time series plots indicated that the stock index series generally exhibited an upward trend for all the nations with that for the Zimbabwe market almost spiraling exponentially which can be attributed to the high index values which resulted from hyper-activity on the money due to availability of cheap money which needed to be quickly disposed off before it lost value.

A heavy presence of tail dependence was revealed amongst all the possible stock market pairs, and these were heavily right and left tailed. The supremacy of upper tail dependence over lower tail dependence indicated that the stock markets were more prone to gain together than lose together. Hence cross market diversification could be of significant use as a means of portfolio risk reduction within these SADC stock markets for potential investors. This evidence of a high likelihood for right extreme co-movements is of profound importance to stockbrokers as they advocate for regional networking or partnerships amongst themselves, thus spearheading the much desired regional integration on informed grounds rather than on geographical accessibility only. This also cemented the decision of disregarding elliptical copulas in the modeling of the dependence amongst these stock markets, by showing that the data was asymmetrically rather than linearly dependent.

The most appropriate marginal distribution that described the trend on all index data was the Extreme Value (Gumbel)distribution. This indicated the appropriateness of the Gumbel copula in describing the data's underlying dependence structure. The Gumbel copula exhibits strong right tail dependence, hence we conclude that the markets tend to boom together, which is an encouraging factor for investors on portfolio diversification.

The most appropriate Archimedean copulas to describe the pairwise dependence structure amongst the five stock markets were the Gumbel and Clayton copulas for the non-parametric and parametric estimations, respectively. Since tail dependence is a copula property, non-parametric estimation had an edge over the parametric estimation in inference due to misspecification errors introduced by the underlying marginal distributions incorporated in the latter. Hence the non-parametric estimation better explained the dependence exhibited between the stock markets. The Gumbel copula explained dependence amongst the stock markets more adequately, from the non-parametric estimation. Hence the markets tend to flourish together, which is an advantage for portfolio diversification though with caution among investors in the region.

5.2 Recommendations

The data had considerable jumps in it due to lack of proper data warehousing and management systems in place, modeling would now involve jump processes and these can be properly executed by Levy copulas. An advantage of modeling dependence through Levy copulas is that the resulting probability law is automatically infinitely divisible. Levy processes are closely related to infinitely divisible laws, indeed, if stock index data is a Levy process, then the transformed index random variable consisting of sums of independent and identically distributed variables i.e (*deviations from the mean*) is infinitely divisible. The one-dimensional marginal distributions of the stock index will be determined by their respective infinitely divisible law which in turn determines the finite-marginal distributions of Levy processes with independent increments.

Real data on stock index is known to be non-stationary, co-integration analysis of the stock markets, with appropriate data transformations such as the wavelet transformation, would be appropriate to ascertain the existence of long run relationships amongst them as this is able to smooth the noise present in the data without loss of generality. This is paramount in tracking the co-movements between stock markets which is integral in the viability of the integration of the markets using dependence structures alone.

Fluctuations of exchange rates are a major factor in the determination of events transpiring at stock markets as investors respond to them in a bid to mitigate their portfolio risks. Hence it would also be important to study the co-movements of these rates with stock index to avert major losses in the event of a crush on stock markets.

Both parametric and non-parametric estimations unveiled the significant presence of heavy-tailedness within the data. In as much as upper tail had an edge over lower tail dependence, the heavy presence of both cannot be ruled out, hence we suggest that a semi-parametric estimation be undertaken and further that mixture models be considered so as to capture all the dependence structure amongst the stock markets.

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Stock Index Data

Data	BDI	MDI	JSE	LuSE	ZSE
1	2277.240	221.00	7977.99	385.4459	1.594776e+02
2	2278.050	221.00	8081.24	385.4459	1.631229e+02
3	2275.370	221.00	8047.78	357.2019	1.675219e+02
4	2276.420	220.00	8278.55	345.6653	1.699970e+02
5	2276.830	220.00	8292.07	368.5076	1.691915e+02
6	2276.830	220.00	8139.31	347.9726	1.709605e+02
7	2284.060	220.00	7958.38	385.9407	1.692333e+02
8	2246.280	220.00	7818.13	385.9492	1.736879e+02
9	2247.720	219.00	7842.25	385.9507	1.879935e+02
10	2262.090	219.00	7761.55	385.9507	1.853284e+02
11	2262.090	219.00	7746.15	385.9507	1.830430e+02
12	2253.750	216.00	7361.15	385.9045	1.854828e+02
13	2264.710	214.00	7659.39	385.2570	1.873814e+02
14	2272.170	214.00	7644.95	385.2570	1.877273e+02
15	2272.680	215.00	7722.27	385.2570	1.885669e+02
16	2270.050	215.00	7685.49	385.4878	1.888208e+02
17	2266.773	216.00	7564.89	385.4878	1.918069e+02
18	2259.233	215.00	8203.53	386.1970	1.960837e+02
19	2257.597	215.00	8110.68	386.2055	2.016768e+02
20	2254.760	208.00	8108.52	386.1884	2.084936e+02
21	2254.760	208.00	8165.31	386.1884	2.119871e+02
22	2267.087	208.00	8418.70	386.1799	2.100314e+02
23	2265.562	208.00	8580.62	386.1799	2.058767e+02
24	2250.378	208.00	8716.22	394.7254	2.046083e+02
25	2249.961	207.00	8717.26	393.8109	2.105432e+02
26	2250.410	207.00	8868.91	393.6283	2.276830e+02
27	2240.252	207.00	8551.68	393.5226	2.547082e+02
28	2244.765	207.00	8452.76	391.9557	2.599909e+02
29	2233.721	208.27	8756.00	391.3400	3.306927e+02
30	2233.837	208.27	8607.91	391.7800	3.396671e+02
31	2239.959	205.15	9072.60	387.0894	6.137135e+02
32	2240.316	205.15	8990.69	387.0894	6.402045e+02
33	2244.354	201.60	9041.15	387.0894	6.617267e+02
34	2244.354	201.60	9125.26	386.7326	6.749798e+02
35	2260.965	204.19	9260.65	386.7326	7.436306e+02
36	2261.071	204.19	9098.55	369.3097	7.447678e+02
37	2262.101	204.19	9146.28	369.9437	7.490640e+02
38	2258.339	204.19	9190.66	369.7021	7.546040e+02

39	2259.601	204.19	9357.32	387.4880	6.962040e+02
40	2281.452	204.19	9210.38	387.2519	6.710945e+02
41	2289.398	210.46	9313.98	387.0157	6.621380e+02
42	2301.770	210.46	9303.96	387.0157	6.611143e+02
43	2300.952	210.46	9498.09	387.2519	6.859569e+02
44	2301.513	210.46	9631.88	384.7035	7.312187e+02
45	2301.513	210.46	9453.31	384.9396	7.326524e+02
46	2305.164	207.24	9421.57	384.9396	7.330598e+02
47	2340.983	207.32	9156.39	380.8896	6.479398e+02
48	2346.921	207.32	9137.34	380.8896	6.412812e+02
49	2345.230	207.32	9147.97	378.9956	6.512062e+02
50	2373.817	207.57	9807.18	379.5000	6.569470e+02
51	2387.376	217.27	9528.51	381.9000	6.153359e+02
52	2389.680	217.27	9644.84	372.2800	6.179706e+02
53	2384.004	217.27	9695.88	374.4000	6.162442e+02
54	2390.514	217.27	9813.94	370.9700	6.116176e+02
55	2400.932	222.04	9815.37	388.3700	5.999663e+02
56	2400.932	225.04	9792.21	389.7900	5.912666e+02
57	2404.994	229.03	9827.34	401.1255	5.666219e+02
58	2421.974	227.57	9864.33	401.1255	5.749143e+02
59	2425.723	227.82	9710.91	403.1855	5.969046e+02
60	2432.335	227.82	9700.21	407.5380	6.175093e+02
61	2432.387	227.82	9657.66	407.5380	6.280698e+02
62	2437.572	227.82	9584.51	410.1282	6.752865e+02
63	2450.741	230.87	9601.32	405.2105	7.134523e+02
64	2491.426	229.57	9856.98	413.7200	5.936403e+02
65	2491.263	229.57	9920.04	416.8900	5.658379e+02
66	2497.210	229.57	10229.76	416.8902	4.888265e+02
67	2497.272	231.72	10873.22	427.0098	3.142268e+02
68	2499.498	231.72	10943.40	426.5746	3.204499e+02
69	2508.004	231.72	11009.46	421.8044	4.042915e+02
70	2519.213	231.72	11117.15	427.6465	4.748264e+02
71	2519.213	232.23	11064.74	440.7038	4.708564e+02
72	2520.718	232.23	10862.12	437.9623	4.796762e+02
73	2523.772	231.55	10854.39	432.7946	4.847868e+02
74	2523.772	232.23	10849.25	434.4505	4.848579e+02
75	2553.597	237.54	10817.77	434.4505	4.837764e+02
76	2556.494	237.54	10758.17	417.0719	4.887486e+02
77	2555.865	234.00	10651.10	425.7768	4.868092e+02
78	2624.493	240.94	10821.72	434.2754	4.740820e+02

79	2620.412	243.86	11035.65	436.0574	4.643003e+02
80	2622.900	243.86	11155.59	440.5123	4.570951e+02
81	2628.253	243.86	11096.55	436.0732	4.208555e+02
82	2628.526	243.86	10896.26	436.0732	4.062194e+02
83	2626.687	243.86	10720.93	426.2929	4.016449e+02
84	2625.131	241.05	10757.89	426.2900	3.932847e+02
85	2637.430	241.05	10657.04	426.2900	3.734588e+02
86	2644.626	241.05	10736.04	435.2027	3.598904e+02
87	2648.707	241.05	10761.01	429.7256	3.448451e+02
88	2655.360	241.05	10910.07	432.3370	3.411669e+02
89	2642.003	234.05	10545.00	440.2124	3.320920e+02
90	2641.250	234.13	10534.40	444.5649	3.357582e+02
91	2645.760	241.13	10617.35	447.4575	3.414193e+02
92	2658.386	241.38	10605.47	447.4575	3.429614e+02
93	2679.726	241.70	10692.56	447.4575	3.477084e+02
94	2685.974	241.70	10765.55	447.4575	3.475533e+02
95	2677.126	242.02	10890.61	446.9649	3.462120e+02
96	2679.322	243.97	10862.17	446.9649	3.457092e+02
97	2689.077	243.97	10904.04	446.9649	3.477869e+02
98	2731.403	243.97	10825.83	446.9649	3.470693e+02
99	2706.143	243.97	10846.37	446.9649	3.482078e+02
100	2709.196	243.97	10979.56	446.9649	3.446839e+02
101	2718.251	243.97	10979.56	446.9649	3.444479e+02
102	2723.962	243.97	10828.19	446.9649	3.451438e+02
103	2723.887	243.97	10757.16	446.7511	3.423476e+02
104	2728.762	243.97	10764.85	446.7500	3.441484e+02
105	2740.771	248.48	10658.56	459.8319	3.501161e+02
106	2744.163	248.48	10639.00	459.8319	3.644317e+02
107	2757.651	248.48	10655.55	459.8319	3.683421e+02
108	2801.134	264.98	10526.55	461.9796	3.833812e+02
109	2797.726	264.98	10574.77	461.9796	4.160944e+02
110	2805.584	264.98	10413.50	461.9796	4.238998e+02
111	2809.064	264.98	10407.90	466.4480	4.280987e+02
112	2815.302	264.98	10064.81	466.4321	4.352878e+02
113	2817.105	266.26	10127.60	466.6618	4.385937e+02
114	2820.295	266.26	10016.38	466.6618	4.412455e+02
115	2820.096	266.26	10031.19	466.6618	4.424665e+02
116	2820.847	266.26	9808.31	466.6618	4.575388e+02
117	2824.440	267.29	10090.76	470.4661	4.732004e+02
118	2839.660	267.29	9989.20	487.0470	4.795326e+02

119	2863.750	269.88	10200.57	487.5395	4.849334e+02
120	2864.680	280.59	10388.14	499.3830	4.826006e+02
121	2864.740	280.59	10307.22	500.5344	4.932875e+02
122	2862.940	280.59	10308.47	525.5397	5.366106e+02
123	2863.650	280.59	10317.88	525.6726	5.546635e+02
124	2861.780	280.59	10228.78	525.6726	5.613090e+02
125	2862.500	280.59	10191.33	525.6726	5.748589e+02
126	2856.540	280.59	10366.11	525.6726	5.725887e+02
127	2850.290	280.59	10483.79	525.2361	5.666869e+02
128	2852.680	280.84	10419.52	533.9672	5.659477e+02
129	2849.500	281.40	10182.30	563.3960	5.772975e+02
130	2849.890	281.40	10179.44	567.0660	5.725700e+02
131	2850.410	281.40	10109.52	581.7772	5.750737e+02
132	2851.610	281.40	10043.95	593.1277	5.811474e+02
133	2839.760	281.40	9982.09	593.1277	5.895580e+02
134	2841.600	299.11	10188.87	594.0626	5.893065e+02
135	2844.090	299.11	10220.14	604.7871	6.106715e+02
136	2846.990	299.11	10152.05	605.2236	6.185923e+02
137	2845.830	300.75	10129.88	605.2236	6.559589e+02
138	2845.830	300.75	10108.61	605.2236	6.931470e+02
139	2845.830	305.18	10053.47	616.5398	7.268046e+02
140	2852.350	305.18	10092.99	616.5400	7.382564e+02
141	2839.340	306.47	10173.05	620.6915	8.541664e+02
142	2835.380	306.47	10131.11	620.1096	9.385811e+02
143	2835.830	314.68	10018.07	621.0369	9.570818e+02
144	2832.170	316.45	10131.27	621.0369	8.362839e+02
145	2834.160	317.57	9989.22	617.1079	8.583881e+02
146	2836.580	317.57	10072.86	613.7860	8.580148e+02
147	2836.100	317.57	10163.43	613.0988	8.534526e+02
148	2841.490	317.57	10228.52	613.0988	8.779578e+02
149	2838.090	317.57	10245.00	613.0988	9.098522e+02
150	2838.090	317.57	10305.89	613.0988	9.225490e+02
151	2841.060	317.57	10360.19	613.1000	9.148053e+02
152	2841.060	317.57	10433.17	613.0988	9.076970e+02
153	2842.260	317.57	10447.30	613.0988	8.895021e+02
154	2842.260	318.86	10419.49	613.0115	8.915093e+02
155	2836.210	322.82	10227.51	653.8065	8.870980e+02
156	2842.400	322.82	10745.53	668.8871	8.779044e+02
157	2835.590	322.82	10687.33	674.2889	8.656124e+02
158	2832.310	322.82	10881.24	674.2889	8.754769e+02

159	2833.810	322.82	10938.15	678.9182	8.892212e+02
160	2834.010	322.82	11166.09	678.9182	9.196120e+02
161	2834.010	322.82	11042.21	709.7112	9.160422e+02
162	2832.860	323.32	10970.40	698.4331	9.172019e+02
163	2828.880	323.88	11017.60	709.9481	9.262522e+02
164	2836.380	343.91	11160.44	710.1335	8.975810e+02
165	2839.210	353.91	11152.19	710.1428	8.845945e+02
166	2839.210	353.91	11223.91	710.1428	8.933675e+02
167	2842.870	356.33	11244.21	710.1428	8.801530e+02
168	2839.520	356.33	11253.10	709.1733	9.133945e+02
169	2859.520	358.38	11226.82	699.4060	9.257142e+02
170	2859.420	358.38	11238.02	703.4183	9.242440e+02
171	2860.660	358.38	11266.42	708.6804	9.231793e+02
172	2860.660	359.07	11219.67	710.1908	9.126140e+02
173	2861.040	379.87	11412.55	710.1908	9.098221e+02
174	2911.100	415.79	12438.51	745.4250	8.407361e+02
175	2916.350	421.19	12306.22	746.5499	9.128068e+02
176	2888.680	426.03	12800.52	775.3484	1.060920e+03
177	2892.920	426.03	12663.45	775.3484	1.110207e+03
178	2900.820	438.62	12664.89	775.3484	1.153670e+03
179	2891.090	438.64	12497.84	773.9460	1.549771e+03
180	2891.510	439.91	12530.30	773.9460	1.544403e+03
181	2896.830	441.42	12699.81	773.7192	1.942708e+03
182	2903.830	446.88	12957.44	773.8030	2.156404e+03
183	2907.290	445.93	13016.60	781.3379	2.116038e+03
184	2903.190	450.89	13002.21	781.3379	2.056561e+03
185	2896.830	450.89	13046.67	788.2700	2.072306e+03
186	2904.510	450.89	13083.46	793.4037	2.084386e+03
187	2910.830	461.05	13107.50	796.2766	2.214674e+03
188	2910.830	461.05	13086.46	807.5030	2.259719e+03
189	2908.890	461.80	13085.19	800.9309	2.370902e+03
190	2924.820	461.80	13132.42	808.3224	2.276715e+03
191	2929.920	461.80	13089.28	809.3689	2.238965e+03
192	2929.240	461.80	13259.76	809.3689	2.165445e+03
193	2946.110	470.46	13250.01	813.5148	2.147989e+03
194	2941.180	472.98	13375.55	815.3127	2.200731e+03
195	2949.180	473.41	13456.89	817.8605	2.061557e+03
196	2971.310	474.44	13541.47	820.0290	1.940100e+03
197	2992.720	474.44	13508.05	833.5713	2.120563e+03
198	3017.510	474.44	13595.11	831.4384	2.171318e+03

199	3016.380	474.44	13555.05	830.1106	2.191645e+03
200	3016.380	474.44	13703.85	830.1106	2.240745e+03
201	3015.960	474.44	13593.17	833.5389	2.191645e+03
202	3017.780	499.48	13465.53	833.5503	2.421057e+03
203	3021.180	499.48	13297.13	833.5503	2.451393e+03
204	3020.760	508.83	13191.47	833.5503	2.455757e+03
205	3020.760	508.83	13214.11	833.5503	2.473159e+03
206	3021.400	508.83	13176.18	833.5503	2.478034e+03
207	3021.400	517.51	13257.61	833.8038	2.652239e+03
208	3021.670	517.51	13242.36	833.8038	2.722490e+03
209	3021.900	517.51	13216.97	833.8038	2.762223e+03
210	3031.790	508.26	13253.80	828.1236	2.917892e+03
211	3031.790	499.58	13173.39	825.7760	3.008195e+03
212	3055.780	499.58	13019.77	844.9882	3.164215e+03
213	3057.990	499.58	13119.26	844.9882	3.211847e+03
214	3059.980	499.83	13075.65	844.9882	3.237791e+03
215	3067.140	499.90	12967.44	834.5335	3.079945e+03
216	3081.610	502.68	12751.37	851.2037	2.683524e+03
217	3089.850	503.24	12467.30	851.4423	3.007873e+03
218	3092.300	503.24	12555.96	859.4187	3.048051e+03
219	3112.500	519.35	12948.15	872.8940	2.939606e+03
220	3111.870	530.21	13159.53	873.9289	2.901040e+03
221	3117.900	530.21	13087.57	873.7594	2.940021e+03
222	3137.440	531.23	13106.10	873.7594	3.010615e+03
223	3137.440	531.23	13157.21	873.9695	3.013042e+03
224	3158.500	531.23	13097.34	876.3527	3.135557e+03
225	3164.850	531.23	13268.93	894.4500	3.103472e+03
226	3161.400	531.23	13234.00	886.9500	3.142020e+03
227	3172.000	531.23	13418.33	894.8500	2.988457e+03
228	3178.810	531.23	13483.90	886.9500	2.919731e+03
229	3185.650	537.08	13770.34	928.5402	2.858588e+03
230	3197.300	539.39	13787.02	955.2400	2.845275e+03
231	3205.200	539.39	13859.48	958.8300	2.836008e+03
232	3209.650	545.86	13965.33	959.6397	2.836210e+03
233	3229.040	547.13	14087.43	971.9570	2.740972e+03
234	3232.690	547.85	13980.26	969.6939	2.689211e+03
235	3235.000	547.85	14000.44	969.7105	2.638710e+03
236	3235.430	574.68	14001.86	969.6939	2.620935e+03
237	3240.720	577.67	14029.06	977.2279	2.518125e+03
238	3250.830	590.09	14210.74	989.6490	2.363182e+03

239	3256.780	603.76	14345.63	1016.6192	2.528064e+03
240	3265.950	604.32	14311.91	1014.3764	2.653252e+03
241	3267.960	604.32	14313.51	1016.2752	2.774796e+03
242	3275.080	604.32	14278.43	1016.2752	2.852844e+03
243	3267.640	604.40	14103.92	1016.2752	2.869331e+03
244	3272.780	605.67	14047.98	1016.2752	2.878471e+03
245	3275.230	605.67	14065.84	1015.5218	2.866179e+03
246	3275.790	607.45	14302.11	1026.0436	2.772077e+03
247	3276.940	607.45	14515.59	1023.8120	2.819894e+03
248	3286.210	609.20	14671.87	1025.9466	3.020097e+03
249	3286.210	613.29	14779.95	1025.9466	3.078900e+03
250	3286.210	618.83	14720.93	1027.4307	3.168596e+03
251	3279.930	632.25	15058.32	1072.1831	4.204283e+03
252	3287.690	632.25	15120.49	1074.2529	4.132992e+03
253	3303.300	633.76	15219.56	1075.4882	4.065412e+03
254	3311.870	633.76	15143.64	1073.7931	4.005954e+03
255	3308.380	632.49	15326.44	1080.9856	3.979448e+03
256	3312.720	634.56	15430.80	1080.9856	4.008110e+03
257	3327.040	634.56	15430.80	1080.9856	4.028753e+03
258	3338.570	633.76	15464.37	1080.9856	4.124058e+03
259	3339.250	644.27	15736.91	1083.7979	4.257771e+03
260	3339.250	644.27	15709.78	1124.6488	4.462214e+03
261	3334.910	645.14	15716.67	1094.9303	4.500174e+03
262	3338.730	645.14	15515.28	1153.6619	4.207981e+03
263	3372.670	649.82	15473.60	1188.8499	4.207981e+03
264	3371.820	649.82	15625.59	1190.4685	3.927304e+03
265	3353.470	649.82	15675.66	1177.1951	3.861305e+03
266	3358.900	649.82	15435.07	1230.1023	3.859197e+03
267	3359.000	648.54	15382.89	1230.1023	3.813008e+03
268	3367.470	648.58	15298.74	1230.1023	3.813008e+03
269	3374.170	652.46	15266.42	1165.9294	3.813008e+03
270	3381.900	652.46	15300.52	1234.6566	3.812916e+03
271	3308.970	652.46	15414.01	1234.6566	3.812916e+03
272	3380.860	652.46	15646.47	1234.6566	3.812916e+03
273	3392.660	657.54	15844.50	1223.7477	3.807861e+03
274	3457.330	657.54	15939.88	1253.4383	3.807861e+03
275	3442.780	657.54	15849.37	1259.1434	4.061300e+03
276	3444.910	657.54	15890.19	1265.3362	4.546762e+03
277	3426.150	657.54	15940.18	1239.0494	4.744675e+03
278	3427.520	660.63	15984.66	1237.7834	4.637166e+03

279	3427.100	660.63	16148.66	1271.6178	4.533247e+03
280	3430.560	660.63	16372.93	1274.7928	4.470278e+03
281	3430.740	660.73	16472.83	1276.8626	4.449166e+03
282	3442.110	660.73	16227.98	1261.9198	4.425309e+03
283	3442.540	662.95	16140.45	1261.9198	4.485612e+03
284	3453.840	667.38	16392.84	1258.7128	4.610507e+03
285	3466.000	667.38	16453.02	1271.9175	4.746681e+03
286	3467.280	667.38	16638.38	1278.6570	5.172503e+03
287	3478.470	667.38	16661.66	1280.8038	6.677823e+03
288	3469.430	667.38	16694.07	1293.7300	7.160853e+03
289	3475.450	669.38	16589.96	1293.7300	7.596245e+03
290	3481.090	669.38	16298.13	1293.9389	7.590075e+03
291	3484.100	669.38	16136.26	1291.7338	7.634907e+03
292	3483.530	669.38	16303.90	1293.9803	8.596486e+03
293	3511.170	669.38	15935.30	1287.2408	8.224071e+03
294	3511.170	669.38	15661.79	1275.5944	8.638965e+03
295	3516.340	669.38	15827.31	1295.0860	9.932628e+03
296	3525.510	669.38	15539.60	1311.2972	1.028100e+04
297	3524.040	669.38	15834.22	1311.2972	1.122802e+04
298	3524.460	669.38	15616.02	1311.2972	1.401287e+04
299	3533.300	669.38	16442.68	1229.8096	1.273820e+04
300	3531.190	656.57	16249.06	1246.6670	1.463867e+04
301	3534.540	656.57	16433.10	1253.0833	1.370466e+04
302	3532.100	656.57	16558.07	1252.8763	1.452295e+04
303	3530.500	656.57	16566.94	1252.9148	1.547760e+04
304	3535.450	682.26	16789.35	1211.4961	1.990054e+04
305	3530.860	682.26	16999.57	1211.5168	1.956058e+04
306	3540.620	682.26	17394.38	1215.6923	1.968261e+04
307	3540.620	682.26	17294.72	1213.4458	1.893039e+04
308	3532.150	682.40	17127.19	1254.7667	1.831361e+04
309	3561.250	682.54	17288.02	1254.3000	1.671074e+04
310	3545.220	682.54	17287.53	1254.3000	1.506146e+04
311	3545.860	683.03	17450.66	1256.7490	1.629407e+04
312	3545.540	693.16	17030.53	1255.0737	1.681681e+04
313	3547.990	693.16	16774.54	1252.8272	1.645823e+04
314	3547.820	696.48	16800.93	1252.7656	1.613917e+04
315	3555.130	696.48	17016.47	1212.4800	1.643022e+04
316	3555.260	697.71	17292.38	1212.4800	1.617482e+04
317	3555.940	687.00	18078.03	1224.1766	1.725741e+04
318	3557.650	687.00	18172.45	1235.4091	1.805572e+04

319	3563.560	693.23	18473.34	1265.0403	2.477675e+04
320	3587.120	694.97	18594.16	1260.9573	2.758757e+04
321	3603.000	694.97	18567.64	1311.4672	2.945654e+04
322	3613.720	696.24	18304.08	1329.2943	3.162037e+04
323	3622.140	697.35	18730.61	1341.2083	3.331766e+04
324	3621.280	697.35	18880.43	1344.3849	3.485663e+04
325	3622.200	697.35	18797.95	1345.4198	3.528134e+04
326	3620.840	697.35	19016.93	1288.3175	3.637956e+04
327	3620.310	697.35	19262.40	1318.0845	4.154589e+04
328	3661.920	707.33	19589.94	1343.8403	4.857734e+04
329	3663.480	707.33	19745.16	1323.6118	4.527630e+04
330	3670.780	707.33	20040.95	1324.6467	4.527630e+04
331	3678.800	707.33	20076.70	1320.4796	4.412402e+04
332	3681.730	707.33	19769.18	1323.2670	4.602254e+04
333	3732.010	707.33	19469.59	1325.0593	4.207127e+04
334	3735.830	707.33	18970.15	1327.5755	4.321939e+04
335	3740.410	707.33	18750.43	1324.7984	4.321732e+04
336	3749.090	752.24	18734.36	1324.7984	4.308478e+04
337	3766.870	746.88	18665.13	1324.8943	4.272384e+04
338	3812.700	757.61	19085.35	1350.9013	3.887401e+04
339	3823.110	757.61	19391.13	1344.3876	3.554570e+04
340	3825.220	687.87	19369.85	1331.7177	3.363364e+04
341	3853.400	757.61	18626.23	1331.7329	3.298347e+04
342	3857.910	757.61	18540.60	1351.9954	3.367428e+04
343	3861.730	757.61	18818.92	1354.3688	3.473269e+04
344	3876.610	757.61	19289.68	1358.0891	3.511223e+04
345	3876.110	757.45	19692.84	1358.1572	3.438631e+04
346	3894.350	757.61	19820.67	1356.0424	3.223653e+04
347	3891.910	757.61	20138.47	1359.9975	3.219161e+04
348	3887.300	757.61	19991.48	1360.0234	3.120752e+04
349	3889.700	785.34	20438.48	1362.0436	3.133843e+04
350	3898.380	802.02	20677.96	1374.1300	3.154122e+04
351	3952.950	853.04	20711.75	1371.8862	3.112954e+04
352	4055.240	853.03	20647.60	1371.8862	3.089085e+04
353	4077.230	863.02	20685.33	1372.2226	3.023083e+04
354	4081.010	875.46	20631.27	1372.2487	2.951975e+04
355	4079.870	892.10	20725.80	1372.0630	2.944875e+04
356	4090.460	892.10	20865.34	1375.6880	3.028786e+04
357	4087.740	892.10	20722.88	1376.5958	3.098307e+04
358	4090.810	957.22	20666.97	1376.4302	3.220175e+04

359	4129.030	957.72	21046.62	1380.3963	3.557906e+04
360	4152.000	1057.02	21394.18	1380.4928	3.414530e+04
361	4161.930	1059.70	21362.71	1380.4928	3.390709e+04
362	4172.260	1081.89	21525.35	1382.8662	3.409228e+04
363	4172.820	1082.71	21822.24	1380.6197	3.379169e+04
364	4177.480	1220.07	21756.90	1380.6197	3.395154e+04
365	4180.210	1220.07	21913.55	1360.4012	3.624364e+04
366	4185.910	1231.49	22024.03	1362.7597	4.555612e+04
367	4211.960	1231.49	21781.46	1362.7597	4.536805e+04
368	4197.200	1234.03	20974.14	1362.6090	4.688870e+04
369	4216.380	1236.72	20652.61	1380.7466	4.620161e+04
370	4225.790	1241.83	20373.33	1388.9921	4.424941e+04
371	4224.370	1237.95	20199.75	1389.8200	4.314143e+04
372	4251.050	1257.59	20358.25	1374.0564	4.394202e+04
373	4248.640	1257.59	20874.86	1374.0564	4.380440e+04
374	4278.600	1271.78	19690.80	1425.1012	4.075811e+04
375	4279.980	1271.78	18380.06	1438.2409	4.380074e+04
376	4279.390	1345.21	19891.03	1444.3955	4.425008e+04
377	4361.430	1394.46	20282.52	1448.8900	4.546823e+04
378	4366.300	1471.12	20676.72	1474.9237	4.668000e+04
379	4387.060	1471.12	20696.66	1474.9237	4.691923e+04
380	4387.170	1485.67	20499.83	1474.9237	4.695716e+04
381	4384.840	1485.67	20310.79	1474.9237	4.655664e+04
382	4400.910	1485.97	20211.50	1473.6948	5.048409e+04
383	4401.100	1485.97	20763.89	1473.7098	5.291806e+04
384	4446.600	1485.97	21358.31	1478.5228	5.830536e+04
385	4452.240	1417.44	21346.57	1475.2120	5.934039e+04
386	4480.550	1425.73	21591.68	1475.0132	6.204196e+04
387	4475.200	1425.73	20963.97	1475.0132	7.016098e+04
388	4480.210	1420.84	20795.60	1475.1500	7.008377e+04
389	4483.830	1469.79	20520.86	1475.1500	6.989307e+04
390	4538.260	1477.64	20179.65	1475.1500	7.155505e+04
391	4539.150	1480.31	20358.87	1474.9300	7.222374e+04
392	4548.420	1480.90	20413.00	1479.4211	7.395031e+04
393	4634.070	1430.79	20827.37	1480.7818	7.992851e+04
394	4622.200	1430.79	20885.57	1482.8666	9.682312e+04
395	4647.240	1581.47	20616.98	1482.8666	1.184216e+05
396	4722.450	1547.14	20778.06	1517.9122	1.372668e+05
397	4730.130	1539.85	20703.01	1527.0978	1.562421e+05
398	4732.280	1530.06	21001.68	1527.0978	1.609704e+05

399	4811.670	1516.78	20793.45	1527.0978	1.676486e+05
400	4820.780	1516.78	20540.08	1564.9700	1.769568e+05
401	4824.640	1548.18	20700.59	1528.9572	1.875602e+05
402	4862.600	1536.12	20770.57	1520.6914	1.875602e+05
403	4854.220	1537.27	20950.18	1520.6914	1.875602e+05
404	4884.050	1537.57	21080.52	1515.9533	1.894061e+05
405	4881.430	1552.12	21385.14	1524.2190	1.822773e+05
406	4869.140	1552.12	21279.32	1531.7400	1.860572e+05
407	4896.430	1552.12	21733.98	1536.2400	1.875087e+05
408	4917.660	1552.12	21990.14	1536.4628	1.940732e+05
409	4908.620	1509.23	21657.09	1536.3510	2.038390e+05
410	4858.770	1509.28	21664.66	1536.3510	1.971278e+05
411	4861.530	1554.17	21767.62	1538.6043	1.980534e+05
412	4881.730	1554.17	21945.34	1538.6043	1.981959e+05
413	4887.380	1604.29	21670.24	1538.6043	1.840579e+05
414	4962.380	1619.99	22051.38	1538.0500	1.836227e+05
415	5027.340	1624.39	22212.57	1540.6000	1.736768e+05
416	5027.340	1624.39	22480.54	1540.7200	1.763913e+05
417	5057.940	1624.39	22438.48	1545.9800	1.775963e+05
418	5064.240	1624.39	22375.73	1546.2000	1.871631e+05
419	4958.490	1624.39	21884.88	1574.5400	1.995965e+05
420	4989.430	1624.46	21054.39	1575.3200	2.035510e+05
421	5058.780	1624.39	21261.70	1591.4100	2.052522e+05
422	5279.010	1641.44	21418.43	1632.8200	2.320137e+05
423	5300.000	1642.59	21627.42	1623.3500	2.720571e+05
424	5299.630	1642.59	21886.40	1582.0600	2.952736e+05
425	5313.770	1642.60	22008.98	1611.9900	3.111748e+05
426	5292.970	1643.74	22162.65	1603.4400	3.331848e+05
427	5311.070	1643.74	22249.72	1592.6800	3.541733e+05
428	6008.100	1614.62	23209.26	1632.8381	4.014958e+05
429	6100.190	1567.09	23337.66	1632.8300	4.198221e+05
430	6124.370	1567.09	23469.79	1632.8300	4.211735e+05
431	6130.300	1567.09	23489.48	1632.8800	4.076511e+05
432	6131.230	1567.09	23212.71	1636.7700	3.711223e+05
433	6088.750	1567.09	23200.00	1680.7800	3.597401e+05
434	6077.230	1567.09	23338.16	1664.3300	3.445281e+05
435	6321.640	1663.94	23600.52	1664.3300	3.276451e+05
436	6321.410	1663.94	23527.62	1680.7808	3.190269e+05
437	6324.240	1693.31	23590.57	1664.0900	3.197202e+05
438	6322.810	1693.31	23950.99	1675.8200	3.264050e+05

439	6138.300	1690.89	24022.61	1706.8713	3.365319e+05
440	6137.920	1690.89	23952.68	1706.8713	3.551353e+05
441	6148.920	1668.84	23684.67	1693.3700	3.810772e+05
442	6142.970	1668.84	23819.21	1664.0890	4.153660e+05
443	6142.970	1668.91	23566.96	1651.2800	4.221128e+05
444	6140.700	1670.92	23674.39	1671.4292	4.241583e+05
445	6149.340	1672.54	23659.44	1697.8600	4.150727e+05
446	6116.800	1686.03	23436.27	1668.3790	4.176067e+05
447	6113.920	1686.03	23188.80	1670.5400	4.159989e+05
448	6113.920	1686.03	23603.55	1695.3600	4.152478e+05
449	6113.920	1687.51	23792.06	1730.0300	4.247165e+05
450	6118.700	1691.13	23899.11	1745.9527	4.474603e+05
451	6122.170	1684.68	23858.28	1750.0200	4.611023e+05
452	6122.950	1685.25	23990.87	1746.7800	4.757635e+05
453	6122.950	1731.85	23809.72	1750.0887	4.789699e+05
454	6119.960	1731.85	23564.36	1760.9000	4.779120e+05
455	6120.880	1726.98	23691.74	1765.0000	4.779120e+05
456	6120.880	1730.14	23949.95	1765.2200	4.979692e+05
457	6118.800	1735.01	24070.92	1766.0400	5.246949e+05
458	6130.170	1735.01	24073.20	1766.0400	5.325081e+05
459	6136.880	1735.01	24191.44	1792.3900	5.470259e+05
460	6137.200	1735.01	23946.47	1792.4000	5.475795e+05
461	6160.650	1751.24	23856.59	1802.6200	5.410128e+05
462	6160.650	1782.52	23760.59	1835.6700	5.312971e+05
463	6158.960	1782.52	23703.51	1825.8200	5.254890e+05
464	6170.080	1782.52	23555.11	1826.0200	5.239288e+05
465	6176.740	1782.52	23711.17	1815.7300	5.199280e+05
466	6171.870	1782.52	23965.91	1815.7400	5.189020e+05
467	6177.340	1785.95	24149.06	1830.5460	5.202746e+05
468	6233.200	1788.82	24137.27	1815.7400	5.135843e+05
469	6215.950	1779.06	24118.23	1830.5841	5.219056e+05
470	6271.290	1788.82	24447.02	1830.5900	5.291654e+05
471	6176.300	1788.82	24825.00	1830.3700	5.315806e+05
472	6195.450	1793.39	24932.27	1836.3400	5.354361e+05
473	6195.450	1769.00	24985.81	1837.6100	5.460687e+05
474	6195.450	1793.39	24915.20	1837.6061	5.698641e+05
475	6196.490	1800.57	24600.96	1836.0388	5.837604e+05
476	6250.060	1800.57	24201.02	1880.5400	5.980575e+05
477	6255.750	1800.57	24261.06	1861.7005	6.333911e+05
478	6255.940	1807.82	24412.47	1861.9300	6.744720e+05

479	6262.620	1816.77	24603.61	1899.8700	7.108085e+05
480	6266.530	1810.01	24535.63	1928.3500	7.500652e+05
481	6266.530	1810.01	24677.47	1927.6200	7.925238e+05
482	6295.090	1811.16	24908.87	1959.5800	8.444230e+05
483	6289.830	1866.79	25155.45	1930.9000	9.185267e+05
484	6291.780	1866.79	24980.44	1961.8155	8.212700e+05
485	6315.940	1857.03	25231.50	1992.9675	7.922217e+05
486	6323.120	1842.40	24920.95	1992.9675	8.482968e+05
487	6359.310	1866.79	25139.78	2040.0100	8.570635e+05
488	6357.640	1938.32	25103.68	1992.7500	8.875708e+05
489	6386.660	1923.72	25572.39	2051.0400	9.721448e+05
490	6388.800	1954.91	25377.29	2051.9400	9.623915e+05
491	6389.320	1950.03	25341.81	2062.5500	9.572725e+05
492	6389.320	1954.91	25481.25	2079.6700	9.582971e+05
493	6514.060	1931.38	25782.38	2128.2300	1.035862e+06
494	6592.620	1958.06	25679.19	2115.3600	1.126561e+06
495	6592.620	1948.30	25605.18	2060.4040	1.149638e+06
496	6951.920	1942.38	25933.81	2060.4040	1.196518e+06
497	6961.570	1952.36	25915.90	2126.0100	1.219058e+06
498	6915.660	1952.36	25596.86	2059.4200	1.259386e+06
499	6920.490	1952.36	25666.81	2062.3200	1.256705e+06
500	6920.490	1952.51	25820.61	2065.4803	1.255058e+06
501	6980.360	1952.51	26023.35	2132.9317	1.251384e+06
502	7050.370	1952.51	26311.42	2132.9317	1.241896e+06
503	7050.370	1963.40	26521.10	2132.9317	1.224222e+06
504	7053.000	1963.40	26511.08	2138.4200	1.240667e+06
505	7083.360	1973.39	26623.23	2037.9900	1.247570e+06
506	7101.260	1981.86	26514.77	2037.9900	1.237510e+06
507	7128.430	1987.33	26306.30	2074.5600	1.245805e+06
508	7169.150	1987.33	26697.70	2138.4400	1.266974e+06
509	7169.150	2002.31	26792.39	2142.1300	1.271250e+06
510	7174.710	2007.30	26932.15	2142.1300	1.298912e+06
511	7179.970	2008.44	26078.30	2082.6590	1.375554e+06
512	7196.840	2012.46	25795.99	2097.3100	1.479886e+06
513	7242.600	2013.33	25336.58	2101.2100	1.500168e+06
514	7265.430	2013.33	25561.12	2101.6400	1.507188e+06
515	7334.730	2013.33	25228.11	2098.0200	1.574206e+06
516	7343.550	2017.46	25280.40	2152.9800	1.697065e+06
517	7336.960	2052.18	25616.63	2098.2300	2.103953e+06
518	7347.210	2063.87	25923.48	2097.8200	2.394511e+06

519	7363.910	2072.54	25853.05	2101.9600	2.468558e+06
520	7409.960	2072.54	25250.06	2101.9800	2.573700e+06
521	7532.400	2088.18	25662.31	2097.7000	2.800044e+06
522	7534.130	2091.05	25839.13	2119.6100	3.438255e+06
523	7542.350	2091.05	26161.53	2119.7800	4.002796e+06
524	7538.370	2101.03	26360.93	2061.3900	5.207685e+06
525	7715.440	2103.23	26654.90	2098.2000	4.867399e+06
526	7725.840	2115.40	26913.62	2102.5500	4.723232e+06
527	7732.520	2117.60	26756.29	2134.1400	4.659633e+06
528	7794.350	2117.60	26839.63	2167.4900	4.288088e+06
529	7801.530	2117.60	26829.80	2135.9000	4.256127e+06
530	7789.050	2117.60	27150.59	2080.8900	3.956926e+06
531	7793.700	2122.62	27267.24	2077.5000	4.026438e+06
532	7806.940	2122.62	27418.88	2077.7000	3.982281e+06
533	7808.510	2126.05	27568.17	2089.0800	4.102737e+06
534	7808.510	2153.34	27525.74	2078.1100	4.303969e+06
535	7789.650	2171.17	27547.85	2078.1800	4.502882e+06
536	7839.900	2168.98	27875.62	2078.1800	4.684355e+06
537	7856.590	2178.97	27638.64	2126.3100	4.784483e+06
538	7862.540	2236.83	27822.27	2143.5900	4.523607e+06
539	7862.540	2244.97	28132.45	2115.8800	4.345209e+06
540	7868.540	2258.12	28379.51	2131.6000	4.268130e+06
541	7879.160	2330.66	28506.72	2229.8600	4.253469e+06
542	7878.680	2341.09	28098.66	2232.7100	4.246742e+06
543	7886.480	2359.77	28367.54	2241.5400	4.224917e+06
544	7886.540	2400.08	28501.70	2241.9700	4.242757e+06
545	7930.380	2403.17	28223.31	2300.4161	4.714403e+06
546	7953.470	2434.58	28076.19	2331.2100	4.967452e+06
547	7961.630	2443.84	28170.60	2321.7400	5.731347e+06
548	7993.090	2457.68	28328.12	2321.3221	5.464873e+06
549	8004.280	2469.26	28278.23	2321.3221	5.330526e+06
550	8004.280	2474.14	28649.70	2344.1000	5.282214e+06
551	8040.340	2476.56	28797.79	2352.4500	5.426939e+06
552	8239.250	2486.24	28161.78	2417.0100	5.588750e+06
553	8240.970	2483.96	28354.41	2402.2500	5.833793e+06
554	8256.750	2640.33	28438.86	2416.3600	6.072865e+06
555	8313.270	2688.66	28094.80	2425.7900	6.753090e+06
556	8462.940	2688.95	28059.27	2425.8000	6.490137e+06
557	8481.350	2700.39	28331.45	2456.7900	6.825855e+06
558	8601.530	2778.57	28642.40	2457.4100	6.893572e+06

```
559 8680.030 2778.57 28629.38 2495.4549 7.085727e+06
560 8695.070 2795.04 29087.49 2505.4600 7.518644e+06
561 8771.210 2856.93 28812.94 2509.5200 8.206780e+06
562 8814.560 2856.93 28637.22 2524.1500 8.764803e+06
563 8864.310 2857.52 28439.40 2505.5900 1.088621e+07
564 8905.920 2862.20 28942.78 2506.7946 1.260095e+07
565 8928.290 2862.20 28837.52 2520.1900 1.381197e+07
```

R PROGRAM FOR DATA ANALYSIS

```
x=read.csv(file=("c:Documents and Settings\\em
+\\My Documents\\EvidenceMatangi\\CopData.csv")

pairs(x)

cor(x,x)

cor(x,x, method="kendall")

skew=mean((x_i-mean(x_i))^3)/(sd(x_i)^3)

kurt=(mean((x_i-mean(x_i))^4)/(sd(x_i)^4))-3

attach(x)

tsplot(JSE,xlab="time",ylab="index")

tsplot(ZSE,xlab="time",ylab="index")

n=565

JB=(n/6)(skew^2+((kurt-3)^2)/4)

L=function(x){2^{(-1/x)}}

{\bf OR}

L=function(x){max((0.01^{-x}+0.01^{-x})^{-1/x})/0.01,0}

U=function(t){(1-0.02+exp(-((-log(0.01))^t+(-log(0.01))^t))
+^{1/t})/0.99}
```

```
Tail Dependence Analysis for the best non-parametric copula to model
the data, in this case the Gumbel copula

(i) Lower tail dependence analysis

s=seq(length=10,0.01,0.1)
G=function(i)exp(-((-log(i))^{28.49}+(-log(i))^{28.49})^{1/28.49})/i
t=c(G(0.01),G(0.02),...,G(0.1))
plot(s,t)

(ii) Upper tail dependence analysis

s=seq(length=10,0.9,0.99)
G=function(i)1-2*i+exp(-((-log(i))^{7.72}+(-log(i))^{7.72}))
+^{1/7.72}/(1-i)
t=c(G(0.90),G(0.91),...,G(0.99))
plot(s,t)
```

NON-PARAMETRIC FORMULAS FOR DETERMINATION OF ARCHIMEDEAN COPULA PARAMETERS FOR THE DIFFERENT PAIRS OF STOCK MARKETS INDEX DATA

$\tau = \frac{\theta}{2+\theta}$ Formula for determining the Clayton Copula parameter θ between pairs of stock markets

$\tau = 1 - \frac{1}{\delta}$ Formula for determining the Gumbel Copula parameter δ between pairs of stock markets

$\tau = 1 - \frac{4}{\theta} - 2 + \frac{4}{\theta} log(exp^{\theta} - 1)$ Formula for approximating the Frank Copula parameter θ between pairs of stock markets

R PROGRAM FOR THE PARAMETRIC DETERMINATION OF THE COPULA

MODELS DESCRIBING DEPENDENCE AMONG THE STOCK MARKET

`attach(x)`

$x_i = c(exp(-exp(-\frac{(X_i - mean(X_i))}{sd(X_i)})))$ where x_i are the uniform distributed statistics for B,M,J,L,Z coming from the cumulative extreme value distribution

Q=data.frame(B,M,J,L,Z) Data frame of the uniform distributed stock market indices vital for the parametric determination of the appropriate copula models

library(copula)

set.seed(1)

dextreme=function(x,location=0,scale=1,log=FALSE)

$+(\frac{1}{scale}) * \exp^{\frac{-(x-location)}{scale}} * \exp^{-\exp^{(x-location)}}$

pextreme=function(x,location=0,scale=1,log=FALSE)

$+\exp^{-\exp^{\frac{-(x-location)}{scale}}}$

qextreme=function(x,location=0,scale=1,log=FALSE)

$+1 - \exp^{-\exp^{\frac{-(x-location)}{scale}}}$

attach(x)

mclay=archmCopula(family="clayton",dim=2,param=2)

mgum=archmCopula(family="gumbel",dim=2,param=2)

myMvd=mvdc(copula=mclay,margins=c("extreme","extreme"),

+ paramMargins=list(list(location=μ_i,
+σ_i),list(location=μ_j,scale=σ_j)))

myMvd1=mvdc(copula=mgum,margins=c("extreme","extreme"),

+ paramMargins=list(list(location=μ_i,

+σ_i),list(location=μ_j,scale=σ_j)))

dat=data.frame(x[,i],x[,j])

loglikMvdc($\mu_i, \sigma_i, \mu_j, \sigma_j$,2,dat,myMvd)

loglikMvdc($\mu_i, \sigma_i, \mu_j, \sigma_j$,2,dat,myMvd1)

mm=apply(dat,2,mean)

vv=apply(dat,2,var)

b1.0=c($\frac{mm[1]^2}{vv[1]}, \frac{vv[1]}{mm[1]}$)

b2.0=c($\frac{mm[2]^2}{vv[2]}, \frac{vv[2]}{mm[2]}$)

a.0=sin(cor(dat[,i],dat[,j],method="kendall")*$\frac{\pi}{2}$)

start=c(b1.0,b2.0,a.0)

fit=fitMvdc(dat,myMvd*,start=start,

optim.control=list(trace=TRUE,maxit=5000))

fit